CFA/VISHNO 2016

Trapped waves in turbulent jets

P. Jordan[a], A. Towne[b], A. Cavalieri[c], T. Colonius[b], V. Jaunet[a], G. Brès[d] et O. Schmidt[e]
[a]PPRIME-CNRS, CEAT, 43 Route de l'Aérodrôme, 86036 Poitiers, France
[b]Caltech, California Boulevard, Pasadena Ca, 91125, USA
[c]ITA, Praça Marechal Eduardo Gomes, 1228-900 São José Dos Campos, Brésil
[d]CASCADE TEchnologies, 2445 Faber Pl., 94303 Palo Alto Ca, France
[e]Caltech, California Boulevard, Pasadena, 91125, USA
peter.jordan@univ-poitiers.fr

Spectral tones observed in the nearfield of isothermal, high-Reynolds-number turbulent jets are shown to be due to trapped acoustic modes. These are confined to the potential core, have negative phase velocity and exist in a frequency band that stability analysis shows to be bounded by a pair of saddle points in the complex wavenumber plane. The physical nature of the modes is clarified by analogy with a soft-walled cylindrical duct. A space-time stability analysis suggests that they correspond to marginally stable global modes.

Spectral tones observed in experimental and Large-Eddy-Simulation data from a turbulent jet at Mach number, $M = 0.9$, are shown in figure 1. The frequency-wavenumber spectrum of figure 2 shows that these correspond to modes with negative phase velocity (three localised signatures in the left half-plane).

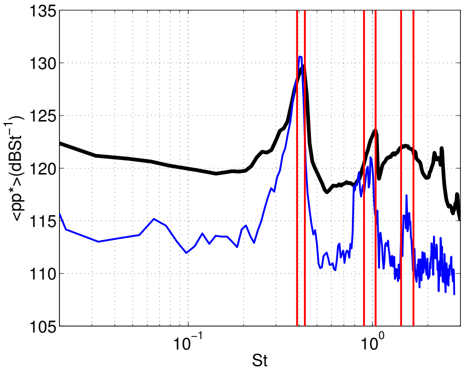

Figure 1 – Pressure spectra of azimuthal mode, $m = 0$, in near-nozzle region, $(x/D, r/D) = (0.1, 0.6)$; black : experiment ; blue : LES

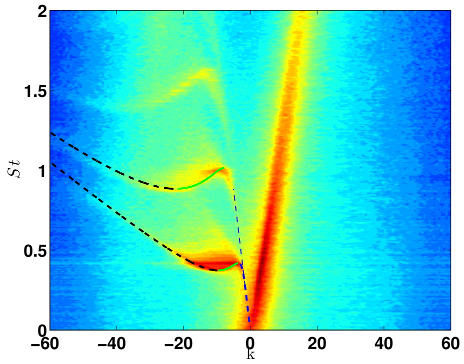

Figure 2 – Frequency-wavenumber spectrum of $m = 0$ pressure at $r/D = 0$ from LES.

We show that the tones are associated with eigenmodes of a cylindrical vortex sheet, which are solutions of the dispersion relation,

$$\mathcal{D}_j(M, \omega, k) = \frac{1}{(1 - \frac{kM}{\omega})^2} + \frac{\Phi K'_m(k\Phi) I_m(k\Psi)}{\Psi K_m(k\Phi) I'_m(k\Psi)}, \qquad (1)$$

where

$$\Phi = \sqrt{1 - \left(\frac{\omega}{k}\right)^2}, \qquad (2)$$

$$\Psi = \sqrt{1 - \left(M - \frac{\omega}{k}\right)^2}, \qquad (3)$$

M, ω and k are, respectively, the Mach number, Helmholtz number and streamwise wavenumber, and I_m and K_m are the modified Bessel functions of the first and second kind.

A criterion can be obtained that determines the conditions under which acoustic duct modes approximately solve the vortex-sheet problem by substituting into equation 1 an (M, ω, k) combination that satisfies the duct dispersion relation. For a soft-walled duct (zero impedance) the criterion is,

$$\left|1 - \frac{kM}{\omega}\right|^2 \gg 1. \qquad (4)$$

Figure 3 shows eigenspectra for vortex-sheet, rigid- and soft-walled ducts and a parallel jet whose shear layer has finite thickness. The inner and outer semi-circles correspond, respectively, to wavenumbers satisfying $(1 - kM/\omega)^2$=1 & 10. The eigenvalues in the left half plane corresponding to the finite-thickness jet, vortex sheet and soft-walled duct almost coincide. At this frequency the first radial mode has cut on (it is on the real wavenumber axis). It is here propagative, with negative phase-velocity and positive group velocity—a space-time stability analysis confirms that this is a k^+ branch. The cut-on point (where the vertical dotted line intersects the real axis) corresponds to a neutrally stable saddle point. The second radial modes are evanescent at this frequency (cf. eigenvalues at $k_i M/\omega \approx 9$). As the frequency is increased these eigenvalues move toward the real axis along the dotted line, and eventually cut on ; the duct eigenvalues continue their trajectory along the real axis ; whereas the vortex-sheet and jet eigenvalues depart into the upper half plane just after crossing the outer semicircle (i.e. they cease to behave like duct modes) ; the departure point corresponds to a second saddle point. The cut-on and cut-off frequencies are plotted as vertical red lines in figure 1 for the first, second and third radial modes. The eigenvalue trajectories on the real axis are plotted in figure 2 : the green line corresponds to the k^+ branch discussed above ; the dashed black and blue lines correspond to k^- branches that form the two saddle points with the k^+ branch. The comparisons confirm that the observed tones correspond to trapped acoustic waves that experience the shear layer of the turbulent jet as a pressure-release surface.

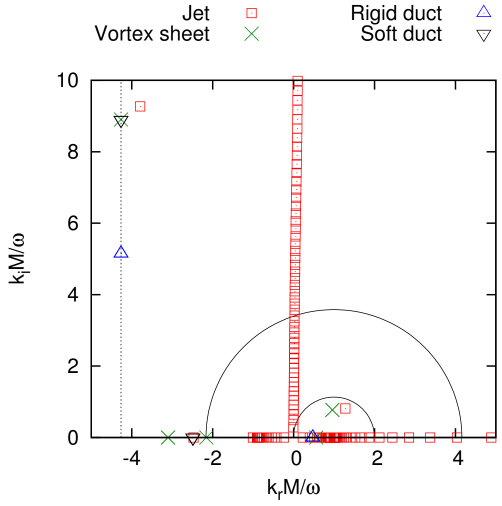

Figure 3 – Eigenspectra for $St = 0.4$.