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# Modal analysis of structures in periodic state

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This presentation focuses on the numerical computation of linear vibrational modes, or Floquet Forms, of mechanical systems in periodic states such as rotating machineries with imperfections or any structures that are in compressive or tensile periodic states. To make our point, we present an original spectral method through the fundamental example of the oscillations of a 2D bi-articulated bar submitted to a periodic compressive load at its end. We show that Floquet Forms generalize the concept of classic modal analysis for structures in equilibrium states. Because of the complexity of the frequency spectrum of Floquet Forms as compared to the classic harmonic modes of vibration, the type of instabilities encountered by periodically-varying structural systems is naturally much richer than systems in equilibrium.

# 1 Context and need

Modal analysis is a key concept in structural mechanics and is today commonly used by engineers in various fields such as Civil Engineering, automotive, aerospace or rotor-dynamics industries. It especially allows to reveal intrinsic vibrational properties of structures in equilibrium states, as well as their local stability, and it is therefore often a necessary step in the design of structures.

Thanks to Floquet theory [1], it should be possible to extend this modal approach to structures in periodic states, i.e. structures with mechanical or geometrical properties that vary periodically with time. Modal analysis of structures in periodic states could be of practical interest for many problems in mechanical engineering including the vibrational behavior of rotating machineries with imperfections, the design of structures submitted to periodic compression or tension loadings or the stability analysis of structures undergoing large oscillations [2]. Surprisingly, to the best knowledge of the authors, this generalization of modal analysis has never been completely and clearly implemented. Due to the conceptual complexity of the vibrational analysis of structures in periodic states, the developed numerical methods mostly focused on dynamic stability [3, 4], neglecting the modal informations that are either ignored, inaccurate or even inaccessible from the computations. As a consequence, the natural link between modal analysis of equilibrium and periodic states yet suggested by Floquet remains largely unknown by the structural engineer and researcher community.

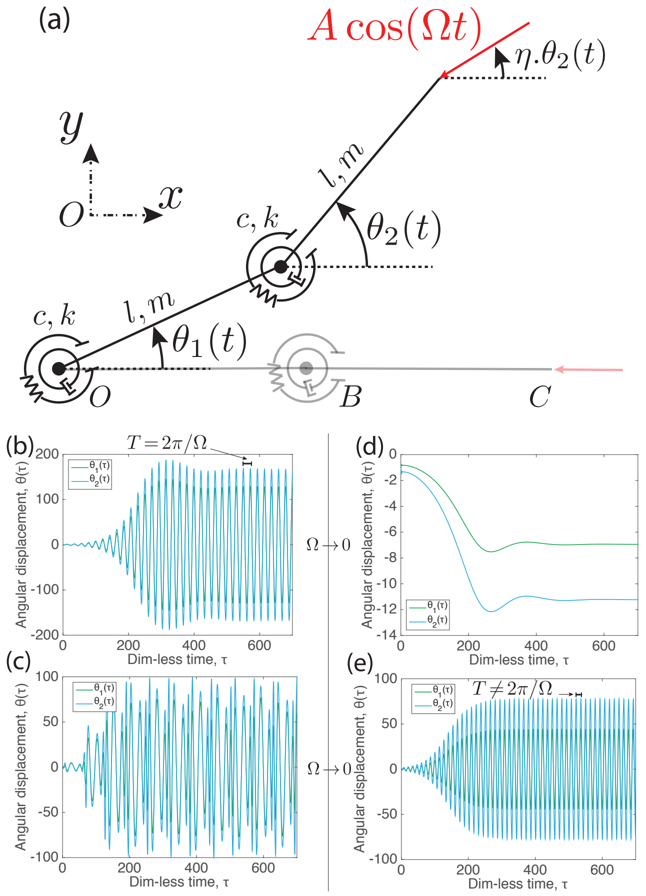


Figure 1 – (a) The system under study is a bi-articulated bar submitted to a compressive load with a period $T = 2\pi/\Omega$ at its end. The load is either periodically conservative ($\eta = 0$, i.e. horizontal force) or non conservative ($\eta = 1$, i.e. following force). (b) Steady-state bifurcation on a $T$-periodic orbit ($\Omega \neq 0$ and $\eta = 0$). (c) Neimark-Sacker bifurcation on a almost-periodic orbit ($\Omega \neq 0$ and $\eta = 1$). (d) Static bifurcation on a new equilibrium state ($\Omega \to 0$ and $\eta = 0$). (e) Hopf bifurcation on a $T$-periodic stationary state with $T \neq 2\pi/\Omega$ ($\Omega \to 0$ and $\eta = 1$).

# 2 Task and Findings

Here, through the archetypal example of a $2D$ bi-articulated bar submitted to a periodic compressive load at its end as illustrated in Fig.1.a, we present an original and accessible spectral numerical framework that generalizes the modal analysis of structures in equilibrium to structures in periodic states. Although apparently simple, this fundamental system encounters all the classic instabilities of 2-dimensional dynamical systems that can be easily sorted depending on the fundamental frequency of the state $\Omega$ and the parameter $\eta$ as illustrated in Fig.1.b-e.

Thanks to a single algorithm based on the sorting of the eigenvalue spectrum of Hill's matrix [5], our method allows to compute Floquet forms (FFs) of periodically conservative or non conservative systems. FFs are almost periodic modal entities reducing to classic harmonic vibrational modes for the particular case of structures in equilibrium states (for a system such as the one illustrated in Fig.1.a, FFs are simply the 2 modes of vibration of the originally straight bi-articulated bar). Like for classic modal analysis, FFs carry the intrinsic vibrational signature of the structure and their spectrum allows to assess the linear stability of the periodic state as illustrated in Fig.2.b-e. In the general case when $\Omega \neq 0$, the almost-periodicity of FFs as well as the dependency of their frequency spectrum on $\Omega$ can cause the periodic state to lose stability and consequently lead to steady-state, period doubling or Neimark-Sacker bifurcations as illustrated in Fig.1.b,c and Fig.2.b,c. In the particular cases when $\Omega \to 0$, FFs reduce to harmonic eigenmodes and the equilibrium state can eventually become statically or dynamically unstable depending on the loading parameter $A$ as shown in Fig.1.d,e and Fig.2.d,e. Like in classic modal analysis, since FFs are first order perturbations of stationary states, they carry the kinematic shape and temporal signature of the bifurcated nonlinear vibrational responses such as illustrated in Fig.1.b-e. Their computation and analysis is therefore crucial for the design of structures in periodic states.

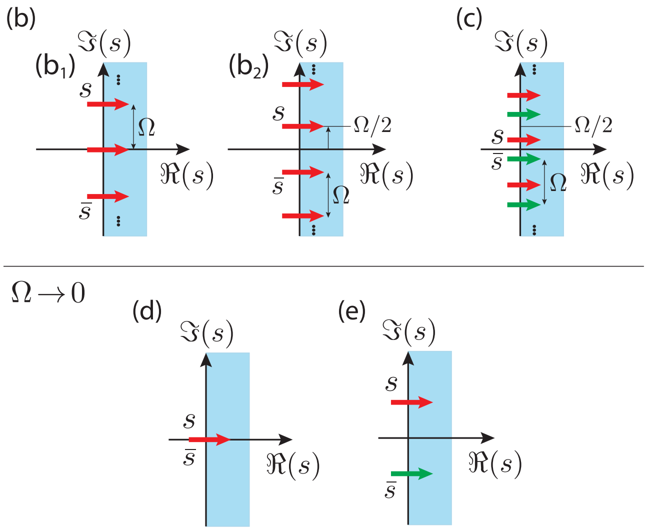


FIGURE 2 – Argand representation of the spectrum $s$ and $\bar{s}$ of the 2 Floquet Forms of the system shown in Fig.1.b-e at the onset of bifurcation. ($b_1$) Steady state bifurcation. ($b_2$) Flip or period-doubling bifurcation. ($c$) Secondary Hopf, or Neimark-Sacker bifurcation. ($d$) Steady-state bifurcation or buckling. ($e$) Dynamic instability or flutter leading to a Hopf bifurcation.

# 3 Conclusions and perspectives

The algorithm presented in this article is a unified method to perform modal analysis of conservative or non conservative systems in periodic states, including equilibria. This unique framework emphasizes the natural link between vibrational behavior of standstill and periodically-varying systems that are common in mechanical engineering. Like for classic modal analysis, this generalized modal approach should allow to design a broader range of structures by computing and analyzing FFs which give better physical insight and understanding on the linear stability of nonlinear dynamical systems. This first study on a simple archetypal 2 dof system will pave the way to the modal analysis of real structures in periodic state with a large number of degrees of freedom. Due to the linear nature of those entities, the interesting properties of classic vibrational modes should remain with FFs which could be promising candidates for modal reduction for structures in periodic states or predictors in nonlinear algorithm such as Newton-Raphson method to compute nonlinear periodic orbits of vibrating structures.

# Références

[1] G. Floquet. Sur la théorie des équations différentielles. Annales scientifiques de l'École Normale Supérieure, 8 : 3-132, 1879.

[2] V. V. Bolotin. The dynamic stability of elastic systems. Holden Day, 1964.

[3] R. Castelli and J.-P. Lessard. Rigorous numerics in Floquet theory : computing stable and unstable bundles of periodic orbits. SIAM Journal on Applied Dynamical Systems, 12 : 204-245, 2013.

[4] J. Zhou, T. Hagiwara and M. Araki. Spectral characteristics and eigenvalues computation of the harmonic state operators in continuous-time periodic systems, Systems & Control Letters, 53 : 141-155, 2004.

[5] A. Lazarus and O. Thomas. A harmonic-based method for computing the stability of periodic solutions of dynamical systems. Comptes Rendus Mécanique, 338 : 510-517, 2010.