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Latent Variable Analysis Based on Hidden Markov Model in Rolling Element Bearing Diagnostics

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In a previous paper, the authors have explored the potential of hidden Markov models in rolling element bearing diagnostics. Based on the Gabor transform, noisy measurements are decomposed in time and frequency, respectively. Then a stochastic model is built in each frequency band. As an extension of the former one, this paper introduces a more exact model which considers a latent variable as a switch corresponding to different distributions and a more complex spectral structure. Therefore, different types of fault based on distinct structures of spectral correlation are separated and estimated. The estimated latent variable carries the information according to the spectral structure. Based on the developed model, a new fault separation scheme is proposed. And the performance is demonstrated by simulated and experimental cases. In simulated case, it reveals expected fault frequencies even in heavy background noise when noise to signal ratio is high. For experimental case, it achieves the superior results to the classical envelop analysis.

1 Introduction

Rotating and reciprocating machines contain various types of rolling element bearings, even a minor loss of material on the surface probably leads to a heavy accident in industrial operation. This is an important reason that fault diagnosis attracts a consistent attention during a long history[1]–[7]. As the rolling surface impacts a local defect, a series of impulse response will be excited. Normally, the impulse response has a short duration which corresponds to some structural resonances in high frequency. In terms of the excitation, we assume that it follows a periodic behavior which carries information of different fault types (e.g. inner race, outer race and rolling element fault). Due to the load distribution, the series of impulse responses are amplitude modulated by the period of passing into and out of the load zone. Therefore, according to the kinematic and geometric parameters, the expected fault frequency could be calculated exactly as shown in [1]. However, in engineering application, these assumptions is of course an idealization : the interval between two adjacent excitations is not strictly periodic due to random slips ; the magnitude of each impulse is not uniform because of the random fluctuations. Even with a small random component, the harmonic structure will be destroyed completely and vanished in high frequency[2]. As an intermediate between stationary and non-stationary signals, cyclostationary process idealy satisfies the feature of rotating machinery. More precisely, rolling-element bearing vibrations are random cyclostationary whose second-order statistics are pseudo-periodic[2, 3].

To follow our related work[4], the former model and assumption should be reminded briefly. The noisy measurement is modelled as a linear combination of a few individual Gaussian distributions along each frequency bin based on the short-time Fourier transform(STFT). The preliminary results demonstrate the potentiality of the proposed model. As mentioned above, the vibration signal in practice is much closer to cyclostationarity. In a short duration, the signal is assumed stationary. To the contrary, in a longer duration, it has a pseudo-periodic feature. As a time-frequency plane, the STFT implies the instantaneous power spectral density which carries the inherent property varing with time instant. This is the reason that the raw measurement is transformed by the STFT which fits well with the basic assumption. As an extension of the former one, this paper introduces a more exact model which considers a latent variable as a switch corresponding to different distributions and a more complex spectral structure.

The rest of the paper is organized as follows. Section 2 presents the assumption and the mathematical model. In section 3, a new fault separation scheme is proposed. After that, the performance of proposed method is analyzed and compared with the classical envelop analysis in section 4. Finally, the conclusion is made in section 5.

2 Spectral mixture model and its assumption

Let denote the signal of interest x(t) which is contaminated by an additive background stationary noise n(t). Then the noisy measurement y(t) is modelled as :

$$y(t) = x(t) + n(t). \tag{1}$$

As discussed before, the vibration signal is a second-order cyclostationary process. The time-frequency plane reveals the fault signatures in both time instant and frequency bin. The STFT of signal x(t) over a time interval of length N_w is defined as :

$$STFT_x(k, f_b) = \sum_{n=kR}^{kR+N_w-1} w_k[n] \cdot x[n] \cdot e^{-j2\pi f_b n_k/N_w} \tag{2}$$

where $\{w_k[n]\}_{n=0}^{N_w-1}$ denotes a positive and smooth N_w–long data–window which shift R samples (from 1 to N) to truncate a segment of x(t) at times kR, ..., kR+N_w-1, then let f_b denote the frequency bin index and n_k denote a local variable (from 0 to N_w-1) related to the time instant k.

In the following model, the phase information will play a crucial role. It is required to correct all the segments to the beginning of the signal, at time instant t = 0. This phase correction also has the same function of the Gabor transform, denoted by :

$$X_G(k, f_b) = STFT_x(k, f_b)e^{-j2\pi kRf_b/N_w}. \tag{3}$$

$X_G(k, f_b)$ means the "instantaneous complex envelope" of signal x(t) in a narrow frequency band Δf centered on f_b and sampled at time index k. The squared magnitude of $X_G(k, f_b)$ reflects the energy flow which is mapped by time index k and frequency index f_b centered in a narrow frequency band Δf.

Hereafter, the measurement y(t) is represented by a linear combination of components with different spectral covariance matrices :

$$Y_G(k, f_b) = \zeta(k)X(k, f_b) + N(k, f_b) \tag{4}$$

where $\zeta(k)$ denotes the proposed latent variable with Bernouilli distribution $\zeta(k) \sim Bernouilli(\pi)$:

$$p(\zeta(k)) = \pi\delta(1 - \zeta(k)) + (1 - \pi)\delta(\zeta(k)). \tag{5}$$

Here $\zeta(k) = 0$ means noise only and $\zeta(k) = 1$ indicates the signal of interest.

Following the assumption above, let deduce the conditional probability :

$$p(Y_G(k, f_b) \mid \zeta(k)) \sim \boldsymbol{CN}(\boldsymbol{0}, C_n + \zeta(k)C_x) \tag{6}$$

where ***CN*** denotes the circularly-symmetric complex normal distribution with the unknown covariance matrices C_n and C_x corresponds to the noise and the signal of interest, respectively. Then the marginal probability function can be written as :

$$p(Y_G(k, f_b)) = \sum_{\zeta(k)} p(Y_G(k, f_b) \mid \zeta(k))p(\zeta(k)) \tag{7}$$

The posteriori probability can be denoted by :

$$p(\zeta(k) \mid Y_G(k, f_b)) = \frac{p(Y_G(k, f_b) \mid \zeta(k))p(\zeta(k))}{p(Y_G(k, f_b))} \tag{8}$$

Hereinbefore, the Spectral mixture model and its assumption are demonstrated. To estimate these parameters of Gaussian densities, the maximum-likelihood parameter estimation is employed as mentioned in [4].

3 Proposed fault separation scheme

In this study, we propose a new fault separation scheme based on the spectral mixture model. The detailed steps are as follows :

1. Transform the raw signal into time-frequency domain using the phase-corrected STFT (see Eq.3), it shows the "instantaneous complex envelope" of signal x(t) in a narrow frequency band Δf centered on f_b and sampled at time index k.

2. Assume the noisy measurement is a linear combination of components with different spectral covariance matrices (see Eq.4). The proposed spectral mixture model considers a latent variable as a switch corresponding to different distributions and a more complex spectral structure.

3. The EM algorithm is employed to estimate the probability of latent variable and the corresponding parameters of different components.

4. Based on the estimated parameters and probability of latent variable, the underlying information can be separated (as shown in following section). An indicator of latent variable is calculated and therefore the expected fault frequency is shown.

4 Numerical and experimental validations

This section contains two parts corresponding to numerical and experimental validations. The first one aims to demonstrate the ideal performance with known parameter values, especially, when the noise to signal ratio reaches a high value. The other one attempts to show the effective performance with real data and the comparison with the classical envelop analysis.

4.1 Numerical validation

The first part presents numerical results. A synthetic signal is generated on the same condition as described in [4]. Figure 1 shows the spectrogram of the raw signal when the noise to signal ratio is 6 dB. With $N_w = 2^8$ long hanning-window, the likelihood ratio in logarithm and estimated $\zeta(k)$ are demonstrated in Figure 2 and 3, respectively. From the likelihood ratio and estimated $\zeta(k)$, both of them demonstrate the expected time intervals as shown in Figure 1. To see clearly the corresponding spectral structure, the estimated C_n and C_x are indicated in Figure 4.

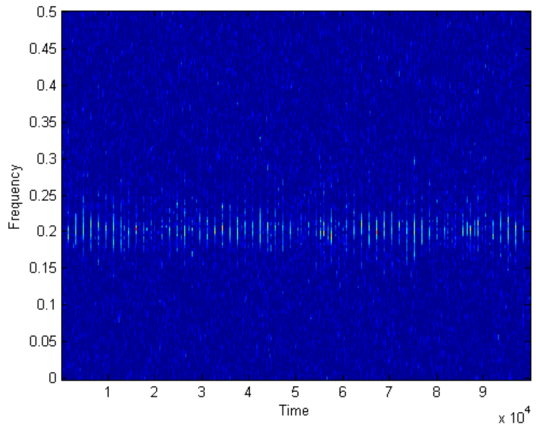

FIGURE 1 – Spectrogram

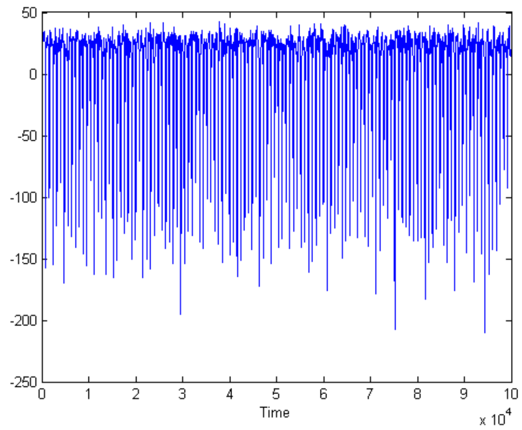

FIGURE 2 – Log likelihood ratio

4.2 Experimental validation

To demonstrate the effective performance of the proposed fault separation scheme, three different types of fault are tested. The tested data is supported by the Vibrations and Acoustics Laboratory of the University of New South Wales (Sydney). The system of the test-rig is a one-stage gearbox with primary and secondary shafts supported by ball bearings, with bearing characteristic frequencies in Hz listed in Table 1.

The first tested signal is denoted by an inner race fault. Figure 5 displays the logarithmic likelihood ratio in frequency domain and the corresponding envelope spectrum of the full bandwidth signal is shown as comparison in

Table 1 – Bearing characteristic frequencies (Hz)

Primary shaft rotation speed	10
Ballpass frequency, inner race (BPFI)	71.10
Ballpass frequency, outer race (BPFO)	48.90
Fundamental train frequency (FTF)	4.08
Ball (roller) spin frequency (BSF)	26.11

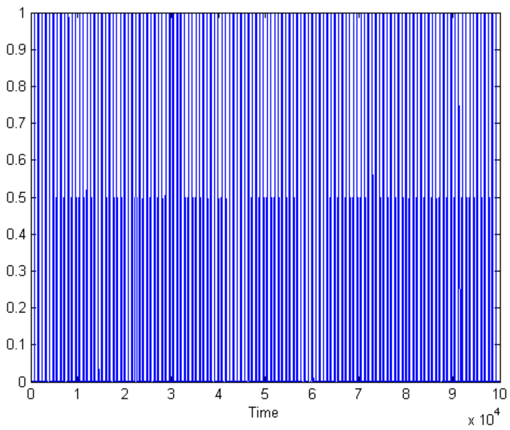

Figure 3 – estimated $\zeta(k)$

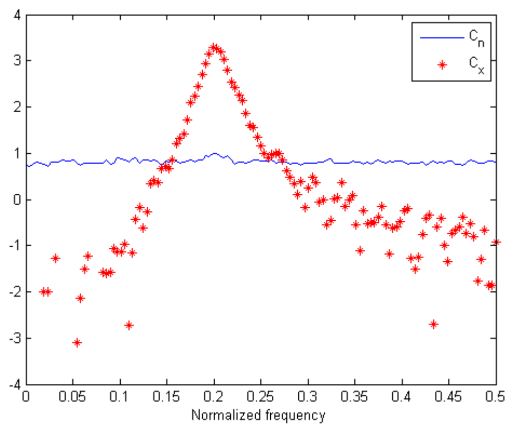

Figure 4 – estimated Log C_n and C_x

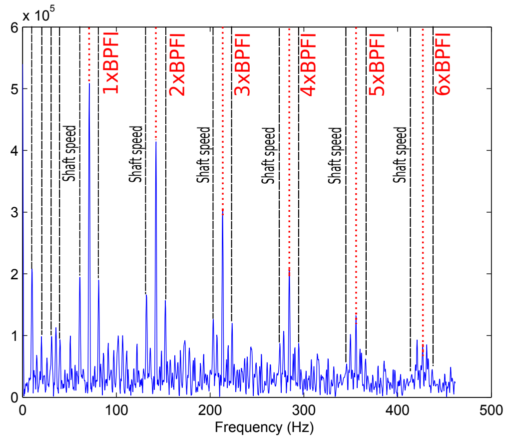

Figure 5 – Log likelihood ratio in frequency domain

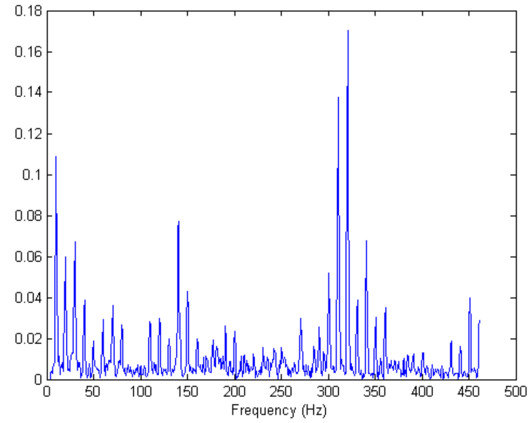

Figure 6 – Envelope spectrum

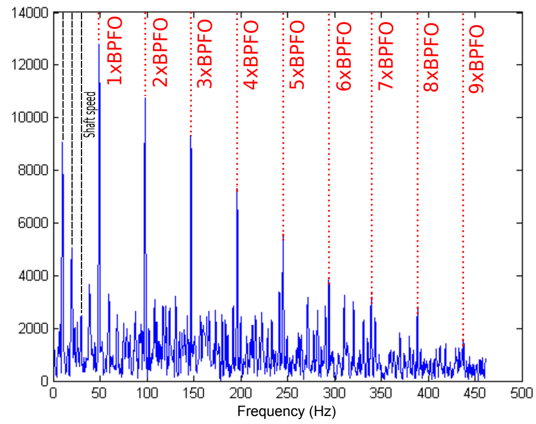

Figure 7 – Log likelihood ratio in frequency domain

Figure 6. It is evident that Figure 5 reveals the expected harmonic structure of inner race fault.

The second analysed signal is denoted by an outer race fault. The estimated logarithmic likelihood ratio and corresponding envelope spectrum are displayed in Figure 7 and 8, respectively. The suspected harmonics of BPFO are presented clearly, and meanwhile there are some harmonics of shaft speed in the low frequency.

The last tested signal is denoted by a ball fault. Figure 9 and 10 display the logarithmic likelihood ratio and corresponding envelope spectrum, respectively. It is noted that for the ball fault case there are harmonics of BSF (with dominant even harmonics of BSF) surrounded by modulation sidebands at cage speed (FTF).

All of these cases illustrate the superiority of the proposed fault separation scheme compared with the classical envelop analysis.

5 Conclusion

This paper is an extention of previous work which improves the former model and assumption. To address the problem of fault diagnosis, a more realistic model was built which considers a latent variable as a switch corresponding to different distributions and a more complex spectral structure. The proposed spectral mixture model contains

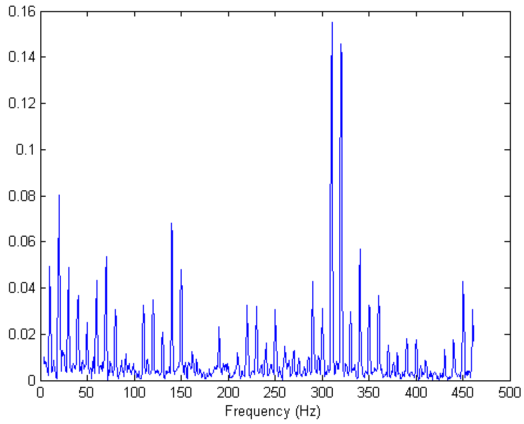

FIGURE 8 – Envelope spectrum

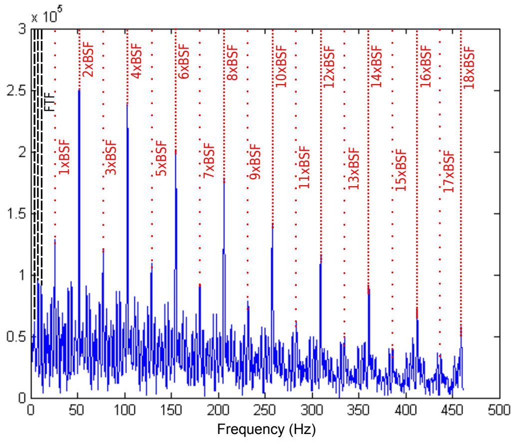

FIGURE 9 – Log likelihood ratio in frequency domain

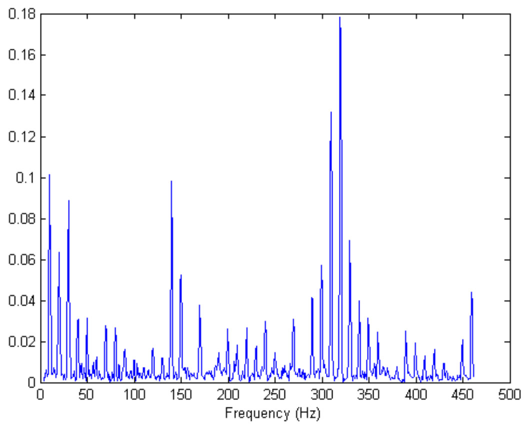

FIGURE 10 – Envelope spectrum

the symptomatic information based on a second–order cyclostationary assumption, yet displaying it in the latent varible and corresponding spectral covariance matrix. Finally, a new fault separation scheme for the vibration signal is proposed and verified with numerical and realistic experiments.

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