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Magnetic-Strung NES avec Récupération d'Energie : Etude Théorique et Expérimentale d'un Nouveau Concept d'Absorbeur de Vibrations Non-Linéaire

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LE MANS

In the last decade nonlinear vibration absorbers, usually known as Nonlinear Energy Sinks (NESs), have been object of several studies in the field of Nonlinear Dynamics and have led to the investigation of various experimental devices. This work illustrates the theoretical design and experimental realization of a new type of Nonlinear Energy Sink. The mass of the Magnetic-Strung NES is a magnet which is linked to the primary system by means of two strings working transversally whose pretensions are adjustable. The restoring elastic force of the strings is modulated thanks to the magnetic force applied by two magnets suitably located on the primary mass. This way, depending on the distance of the additional magnets, either a purely cubic force or a more complex shaped force may be reached. NES efficiency as an absorber is studied on a harmonically forced 1 degree-of- freedom primary system. The Target Energy Transfer (TET) from the primary system towards the NES is experimentally observed as well as different response regimes like the Strongly Modulated Response. Moreover, the energy harvesting from the vibrating energy of the NES is investigated: the NES mass, made up of a magnet, oscillates into a coil and subsequently creates an electric current. Thus, the vibrating energy of the primary mass is in this way absorbed by the NES and finally converted into electric energy.

1 Introduction

A Nonlinear Energy Sink is defined as a passive vibration absorber which is nonlinearly attached to the primary system to control. The use of a Nonlinear Energy Sink (NES) as a vibration absorber has been object of interest in the field of Nonlinear Dynamics in the last decade as studies have shown that, if compared to the classical linear Tuned Mass Damper (TMD), it could be efficient in a broader frequency range and for a smaller addition of mass to the primary system. It has been shown that the nonlinear attachments can lead to an irreversible energy transfer from the primary system towards the NES, this process is known as Targeted Energy Transfer (TET) or pumping [1–5]. Experimental works [6, 7] have shown that the dynamics which governs this energy transfer phenomenon is a 1:1 resonance capture between the primary mass and the NES mass. An important feature of this kind of systems is that they are able to tune itself to the primary system frequency since they do not posses an own natural frequency because of their intrinsic nonlinear nature. TET under external forcing has been investigated theoretically and experimentally [8] showing that in addition to the steady state constant amplitude response regime, another type of response can arise referred to as Strongly Modulated Response (SMR).

Energy harvesting has recently known an increasing importance in many fields: recovering energy from excess heat, electromagnetic waves, and ambient vibration, etc. Vibration energy harvesters use the principles of a spring–mass–damper system that is excited by the external environment. The external vibration causes the mass to oscillate generating mechanical energy. Transduction methods are employed to couple the oscillating mass to an electrical circuit for conversion of mechanical energy to electrical energy. Electromagnetic inductance, capacitance and piezoelectric elements are some of the most common transduction methods. Examples of electromagnetic vibration energy harvesters can be found in [9–11].

In this paper the two research fields, NES and energy harvesting, are combined. The experimental study of a NES prototype with energy harvesting is presented. By means of a magnetic force, the nonlinear force between the NES and the primary system can be adjusted and shaped to test different configurations.

2 The model

In Fig.1 the schematic of the NES is shown. The primary system is the linear oscillator (LO) composed by the mass M, spring K and damper C. The primary system is harmonically forced by the base motion X_e. The NES is composed by the little mass m which is attached to the primary system by means of two flexible strings $\overline{AB}$ and $\overline{BC}$. The strings act as elastic elements and are responsible of the force exchanged between the LO and the NES. The force acting on the mass m as a function of the displacement y can be approximated by the expression:

$$F_{strings} = \frac{T_0}{L}y + \left(\frac{EA}{2L^3} - \frac{T_0}{2L^3}\right)y^3 + O(y^5) \quad (1)$$

Where T_0 is the pretension of the strings, i.e. for $y = 0$, E is the Young's modulus, A the section of the strings. Then the restoring force caused by the strings deflection is composed by a term linearly proportional to the displacement, a term proportional to the cube of the displacement and higher order terms which will be neglected. From eq.(1) we can identify a linear stiffness $K_1 = \frac{T_0}{L}$ and cubic stiffness $K_3 = \left(\frac{EA}{2L^3} - \frac{T_0}{2L^3}\right)$. Both terms are a function of the pretension T_0 which is a parameter we can control in experiments. This is a significant advantage the use of the strings appears to have. Also one should notice that the linear term is a function of the only pretension and ideally if the pretension is zero the restoring force would be a cubic (and higher order terms) function of the displacement.

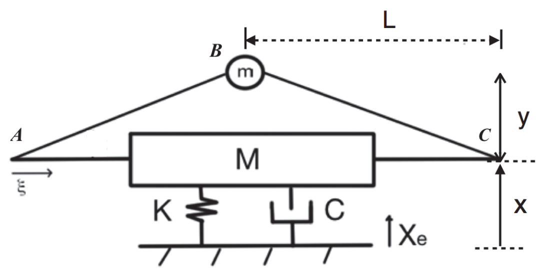


Figure 1: Schematic of the Strung-NES. View from the top. $\overline{AB}$ and $\overline{BC}$ are the strings which act as elastic elements.

Defining the absolute displacement of the NES mass as $z := x + y$, the equations of motion of the system can be written as:

$$\begin{aligned} M\ddot{x} + C\dot{x} + Kx + K_1(x-z) + K_3(x-z)^3 + C_1(\dot{x}-\dot{z}) &= KX_e + C\dot{X}_e \\ m\ddot{z} + C_1(\dot{z}-\dot{x}) + K_1(z-x) + K_3(z-x)^3 &= 0 \end{aligned} \quad (2)$$

An important feature an NES should have is the absence of a natural frequency. It would allow the NES to tune itself to the primary system and to be efficient as an absorber over a broad range of frequency. This characteristic could be reached by the absence of the linear stiffness. As previously mentioned the linear stiffness is zero if the pretension is zero. However, we tried to find an alternative solution in order to study the effect of the linear term and to have a better control on the shaping of the force.

3 The magnetic counterbalance

As NES mass we used a magnet because of the electromagnetic harvesting that will be explained further in the paper. This fact has been exploited as follows for a second purpose. In order to counteract the linear term and have a pure cubic force-displacement relation we put two magnets on each side of the NES magnet. The aim is to obtain a magnetic force that could counterweight the linear term of the elastic force.

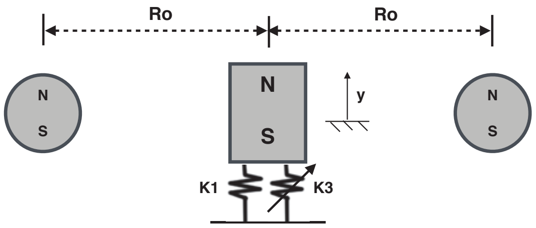


Figure 2: Schematic

The magnetic flux density or **B**-field at the location $\mathbf{r}_p$ due to a magnet located at $\mathbf{r}_s$ is defined by:

$$\mathbf{B} = -\frac{\mu_0}{4\pi}\nabla\frac{\mathbf{m}_s \cdot \mathbf{r}_{p/s}}{|\mathbf{r}_{p/s}|^3} \tag{3}$$

Where $\mu_0 = 4\pi \times 10^{-7}$ H/m is the permeability of free space, $\mathbf{r}_{p/s}$ is the position vector to the point of interest $\mathbf{r}_p$ with respect to the source magnet position $\mathbf{r}_s$, $\mathbf{m}_s = \mathbf{M}_s v_s$ is the magnetic moment of the magnet located at $\mathbf{r}_s$, $\mathbf{M}_s$ and v_s are the magnetization and volume of the source magnet. The potential energy of the magnet at $\mathbf{r}_p$ in the field generated by the magnet at $\mathbf{r}_{p/s}$ is:

$$U = -\mathbf{m}_p \cdot \mathbf{B} \tag{4}$$

Then the interaction force between the two magnets can be obtained by taking the gradient of equation (4).

By applying equations (3) and (4) to the model shown in Fig.2 we obtain the following expression for the magnetic potential energy [11]:

$$U = -\frac{\mu_0 M_c v_c}{2\pi}\frac{M_o v_o N}{2}\left(\frac{y^2}{(y^2+R_o^2)^{5/2}} - \frac{1}{(y^2+R_o^2)^{3/2}}\right) \tag{5}$$

Then the restoring force is the derivative of (5) with respect of y:

$$F_m = -\frac{\mu_0 M_c v_c}{2\pi}\frac{M_o v_o N}{2}\left(\frac{5y}{(y^2+R_o^2)^{5/2}} - \frac{5y^3}{(y^2+R_o^2)^{7/2}}\right) \tag{6}$$

The expression (6) has a trend as shown in Fig.3. In the vicinity of $y = 0$ and as long as the displacement remains small compared to the distance Ro ($y/R_0 < 0.1$), the magnetic force shows a linear behavior.

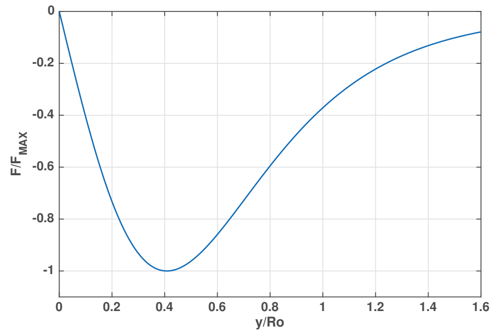


Figure 3: Generic trend of the magnetic force

Thus by taking the derivative of (6) in $y = 0$ we obtain a linear approximation of the magnetic force that seems to be accurate enough for $y/R_o < 0.1$:

$$a := \left.\frac{dF}{dy}\right|_{y=0} = \frac{5C}{R_o^5} \tag{7}$$

$$F \approx ay = \frac{5C}{R_o^5}y \qquad y/R_o << 1 \tag{8}$$

Where C is a constant: $C = -\frac{\mu_0 M_c v_c M_o v_o N}{4\pi}$ depending on the magnetic parameters.

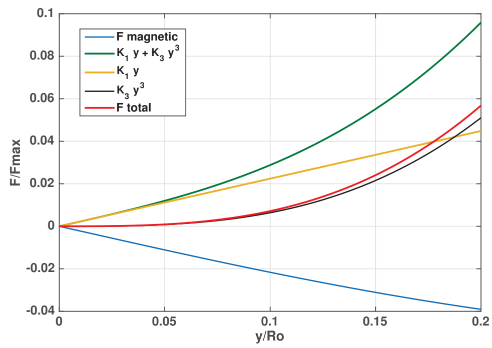


Figure 4: All the forces involved in the balancing: it can be seen as the magnetic force cancels out the linear elastic component and the total final force is actually cubic.

We want the magnetic force to counterbalance the linear component of the elastic restoring force $F_{elastic} = k_1 y + k_3 y^3$. Then by equaling (8) to the linear component of the elastic force we can derive an expression for the distance R_o so that the linear component could be canceled out by the magnetic force.

$$R_o = \sqrt[5]{\frac{5|C|}{k_1}} \tag{9}$$

Fig.4 shows all the forces involved in an example where the distance R_o has been calculated by (9). We can see how the linear component of the elastic force is actually counterweighted by the magnetic force and the total resultant force is purely cubic.

4 The energy harvesting

The method used for the energy harvesting is electromagnetic: the NES mass is a magnet and it oscillates through an electric coil. As the Faraday's Law says, a variation in the magnetic flux means electromotive force:

$$\varepsilon = -\frac{d\Phi_B}{dt} \tag{10}$$

Where ε is the electromotive force and Φ_B is the magnetic flux. The concept combining the NES and harvester is shown schematically in Fig.5: the vibration energy of the primary system flows to the NES and is finally converted into electric energy through the harvester.

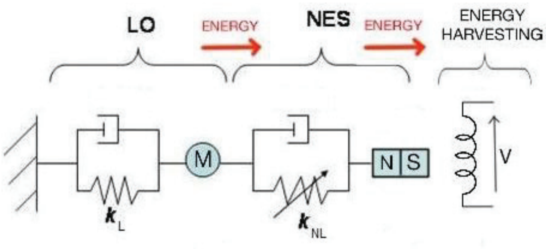


Figure 5: Schematic of the energy flux from the Linear Oscillator to the harvester.

The presence of the harvester creates an electromechanical coupling between the mechanical system and the electrical circuit, schematically presented in Fig.6. It contains the coil's inductance L and the internal resistance R_i along with a resistive load R_L.

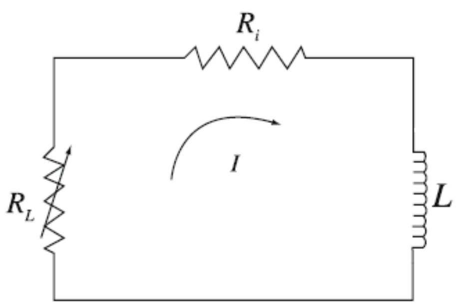


Figure 6: Schematic of the coupled electrical circuit.

The coupled electromechanical system becomes:

$$\begin{gathered} M\ddot{x} + C\dot{x} + Kx + K_1(x-z) + K_3(x-z)^3 + C_1(\dot{x}-\dot{z}) = KX_e + C\dot{X}_e \\ m\ddot{z} + C_1(\dot{z}-\dot{x}) + K_1(z-x) + K_3(z-x)^3 + F_m(z-x) - \gamma I = 0 \\ L\dot{I} + (R_L+R_i)I + \gamma(\dot{z}-\dot{x}) = 0 \end{gathered} \tag{11}$$

We can notice that compared to the initial system (2), in the second equation of system (11) we added the term of magnetic force F_m (whose expression is given by (6)) and the coupling term γI. I is the current flowing into the circuit and γ is a transducer constant that can be derived from Faraday's Law. The third equation is the electrical equation governing the circuit in fig 6. Here the constant γ multiplies the relative velocity between the NES magnet and the coil and their product is a voltage.

5 Experimental investigations

In Fig.7 and 8 the prototype is shown. All the blue and the orange parts have been 3D printed at Duke University. The lower part is a cart sliding on an airtrack that gives the system its one d.o.f. and minimizes the friction. The two sides are connected by two springs respectively to the ground and to the shaker. The NES is a magnet which is placed into a hollow tube and onto a nonmagnetic low-friction slider over the primary mass. The NES is connected to the primary mass by means of two strings which work transversely when the NES oscillates. On the sides of the NES the two outer magnets can be noticed, it is important to highlight that their distance to the NES is adjustable in order to reach the suitable force shape. Finally the coil which the NES mass oscillates through is placed on the primary mass.

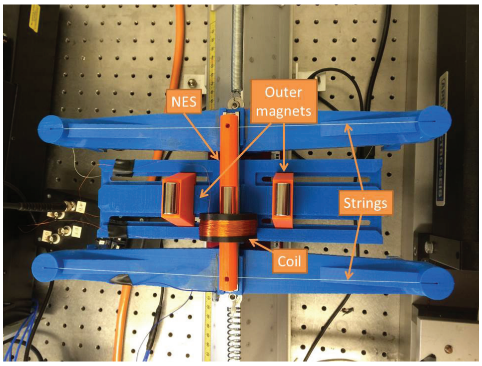


Figure 7: The prototype of the Magnetic-Strung NES.

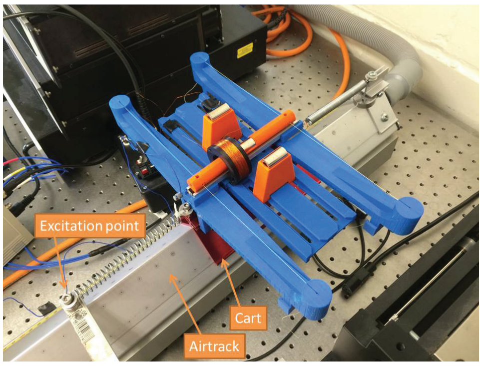


Figure 8: The prototype of the Magnetic-Strung NES. View from the top.

In Tab.1 the modal parameters of the linear primary system are listed along with the mass ratio between the NES and primary system.

$f_0[Hz]$	$K[N/m]$	ξ	$\epsilon = m/M$
5.7	1223	0.036	0.04

Table 1: Modal parameters of primary system.

The electrical parameters have been experimentally measured and are listed in Tab.2.

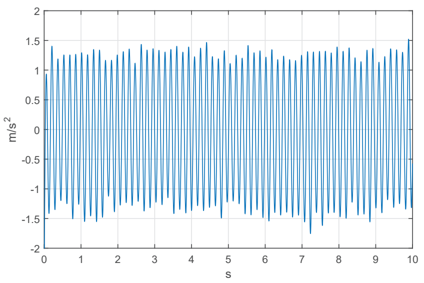
The system was harmonically forced by the base motion at several amplitudes and frequencies. The aim was to observe the types of response the system could exhibit

$L[H]$	$R_L[\Omega]$	$R_i[\Omega]$	$\gamma[Vs/m]$
100×10^{-3}	100	3500	-3.2572

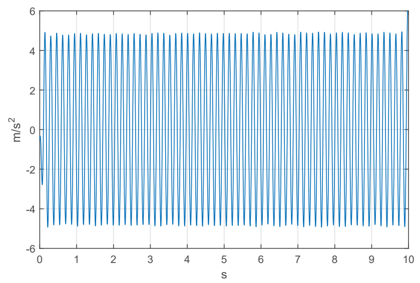
Table 2: Electrical parameters.

and to study its performance in terms of energy absorption and harvesting. The primary mass and the moving base were equipped with accelerometers. The NES motion was monitored by using the signal issued by the coil, indeed that signal can be considered as proportional to the magnet velocity.

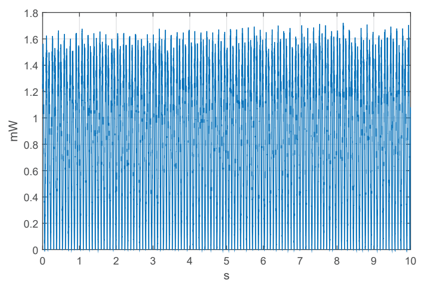
Fig.9 and 10 show the same case of external excitation ($0.6m/s^2$ at $6.2Hz$) for two different configurations of the MS-NES. In the first case the outer magnets were not used, in the second case the outer magnets were placed at a distance that makes the force between the primary system and the NES purely cubic.



(a) Primary mass acceleration.



(b) NES acceleration.

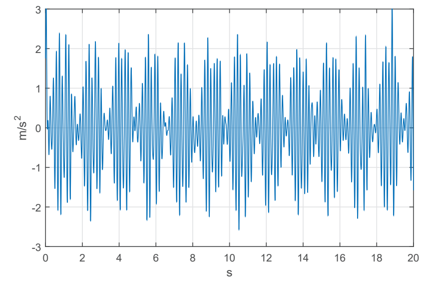


(c) Instant electrical power delivered.

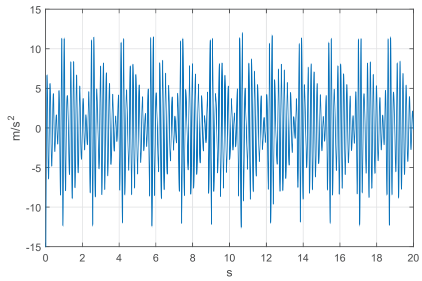
Figure 9: Excitation at $f = 6.2Hz$ without outer magnets.

We can see as for the case in Fig.9 the NES behaves basically as a linear absorber and the response of the system is perfectly steady: the linear component of the elastic force of the strings is present and has a predominant role in the NES behavior.

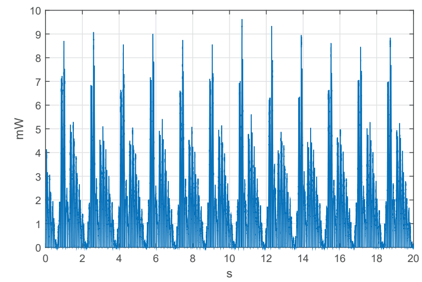
On the other hand, in Fig.10 we can appreciate the benefit of using the outer magnets. In this case the magnet force cancels out the linear component of the elastic strings force and the relation between the primary mass and the NES is purely cubic. The response of the system becomes modulated, not steady anymore but periodic. This kind of response is known in literature as Strongly Modulated Response (SMR) and is typical of nonlinear absorbers [3].



(a) Primary mass acceleration.



(b) NES acceleration.



(c) Instant electrical power delivered.

Figure 10: Excitation at $f = 6.2Hz$ with outer magnets.

6 Conclusions

In this paper the study of a new concept of nonlinear absorber has been introduced and its experimental realization presented. The force between the primary system and the NES is created by the combination of the elastic force delivered by the transverse motion of two strings and an additional magnetic force. The energy absorbed by the NES is converted into electrical energy by means of an electromagnetic transducer. The results have shown as the presence of the magnetic force allows the NES to reach a purely cubic force-displacement relation and to exhibit the sought nonlinear behavior (SMR). This result confirms the importance of having a purely nonlinear force between the NES and the primary system as the presence of a linear component may radically change the global behavior.

Finally the SMR seems to be more favorable in terms of energy absorbtion suggesting that the nonlinearities may be used to improve the energy harvesting.

In summary this study unifies the research fields of nonlinear vibration absorbers and energy harvesting from vibrations showing the advantages of a combined application. An optimization process is still to be done in which the performances of the NES and the harvester would be investigated.

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