CFA/VISHNO 2016

Caractérisation de nombres d'ondes complexes par transformée de Laplace : application aux ondes guidées dans les matériaux poreux

A. Geslain[a], J.-P. Groby[b], A. Duclos[b] et P. Leclaire[a]
[a]DRIVE, Univ. Bourgogne Franche-Comté, 49 rue Mademoiselle Bourgeois, 58027 Nevers Cedex, France
[b]Laboratoire d'Acoustique de l'Université du Maine, Avenue Olivier Messiaen, Cedex9, 72085 Le Mans, France
alan.geslain@u-bourgogne.fr

LE MANS

We present a new method (SLATCoW) for the recovery of complex wavenumbers from porous material guided wave measurements. This method is based on the analysis of the spatial Laplace Transform of the measured normal surface displacement in the frequency domain. The SLATCoW method is applied for the assessment of the experimental complex shear modulus. Besides the recovery of complex wavenumbers from guided wave measurements, the SLATCoW method paves the way to the experimental determination of the viscoelastic parameters of porous materials together with the development of refined models to account for viscoelasticity.

1 Introduction

In surface wave physics, a system is dispersive as soon as one of the characteristic dimensions is similar to the wavelength of the wave propagating in it. To name but a few, wave guides, sub-wavelength resonators embedded in or deposited on a propagation medium, or even the structure itself of a natural compound are some examples of systems presenting dispersive behavior. The main feature of a dispersive system is indeed the dependency of its properties with the frequency. Another feature of dispersive systems, related to dispersion, is the (non-geometrical) attenuation. The Kramers-Kronig relations [3] reflect the relation between the dispersion and the attenuation in such dispersive systems, although the attenuation may belong to different physical processes. Beside the geometrical attenuation, the attenuation of waves can indeed be related to intrinsic loss in a material (heat dissipation, viscosity, scattering, photoelectric effect, etc.) or to the heterogeneity of the system (band gap in photonic/phononic crystals, resonant phenomena, etc.). Measuring both the dispersion and the attenuation is of utmost importance in wave physics and for material science, since it provides insightful information on the tested system allowing the characterization of properties (effective properties) of a material (heterogeneous system).

In general, plane waves in anelastic media are expressed with a wavenumber along the planes of constant phase direction and an attenuation factor along the planes of constant amplitude direction. Assessment of complex wavenumbers (the imaginary part represents the attenuation factor) is then essential. In the framework of elastic guided waves, a few methods have been developed for assessing the attenuation. Some are based on direct measurements of wave amplitude decrease with respect to time[8] or to the propagation distance[9, 2]. These methods are well suited for cases where a unique mode is generated and detected, or at least where isolating one mode is achievable. Other methods have in common to use an optimization process which minimizes the error between a physical model and the measured signals. A good understanding of the involved waves is mandatory to propose a suitable minimization problem and an associated experimental protocol. In geophysics for instance, assuming Rayleigh wave is the only detected mode, the recovery of complex wavenumbers has been used to assess the attenuation associated both to the geometric spreading and to the intrinsic energy dissipation[9, 10, 11]. Note that a multichannel approach has also been used in geophysics to estimate complex wavenumbers with a beamforming techniques with complex propagation constant[12].

In this paper, a method presenting none of these restrictions is proposed. The method makes use of a spatial Laplace transform to achieve the complex wavenumber recovery from guided wave measurements. The acronym is SLATCoW for Spatial LAplace Transform for COmplex Wavenumber recovery. The method is presented and its actual implementation is discussed in the first section. The following section is dedicated to the application of the SLATCoW method for the guided elastic waves in porous materials.

2 The SLATCOW method

Guided wave measurements classically consist in recording the normal displacement (or velocity) on the surface of a flat sample along a line of length L when it is excitated by a line source or a point source. Assuming a time dependance $e^{-\mathrm{i}\omega t}$ (ω is the angular frequency) and neglecting the branch integrals arising from the application of the residue theorem this displacement reads in the frequency domain (ω is dropped for clarity) as the sum of the contribution of each modes :

$$u_n(x) = \sum_{m\in\mathcal{M}} \tilde{u}_n^m \exp\left(\mathrm{i}K^m x\right) \Pi(x-L), \tag{1}$$

where n refers to the normal component of the total displacement $\mathbf{u}(x)$, $\tilde{u}_n^m$ is the complex amplitude of the m-th mode, K^m is the wavenumber of the m-th mode, $\mathcal{M}$ is the set of modes, and $\Pi(x-L)$ is the gate function equal to 1 when $x \in [0, L]$ and equal to 0 elsewhere. These wavenumbers K^m are usually considered purely real, but they are complex. This is particularly the case when the sample is constituted of dissipative materials, but also for some specific modes resulting for example from mode hybridization or repulsion, or in presence of bandgaps due to local resonance of elements constituting the sample material. According to the time Fourier transform convention, the expression of K^m has to be $K^m = k_r^m + \mathrm{i}k_i^m$, with $k_r > 0$ and $k_i > 0$ for Eq.(1) to involve only forward propagating modes. Applying the usual spatial Fourier transform only enables to recover k_r^m. In order to recover both real and imaginary parts of K^m a spatial Laplace transform is applied to $u_n(x)$ denoted $U_n(s) = \int_{-\infty}^{\infty} u_n(x) \exp(-sx)$ with $s = s_i + \mathrm{i}s_r$. This spatial Laplace transform takes the form

$$\begin{aligned} U_n(s) &= \sum_m \tilde{u}_n^m \int_0^L \exp\left((\mathrm{i}K^m - s)x\right) dx \\ &= L \sum_m \tilde{u}_n^m \exp\left((\mathrm{i}K^m - s)L/2\right) \frac{\sinh\left((\mathrm{i}K^m - s)L/2\right)}{(\mathrm{i}K^m - s)L/2}. \end{aligned} \tag{2}$$

The meaning of the spatial Laplace transform of the m-th mode is ensured only if $s_i \geq -k_i^m$, because this ensure the energy decay of the m-th mode (related to $\exp\left((-k_i^m - s_i)x\right)$ in Eq.(2)). The upper half space, the lower bound of which is the maximum value of $-k_i^m$, with $m \in \mathcal{M}$, is the only admissible half space in the complex s-plane and defines the region of absolute convergence of the Laplace transform. Let us notice that the slice of $U_n(s)$ in the complex s plane at $s_i = 0$ exactly corresponds to the usual spatial Fourier transform. Along this line, Eq.(2) reduces to $L \sum_m \tilde{u}_n^m \exp((k_r - s_r - \mathrm{i}k_i)L/2)\,\mathrm{sinc}((k_r - s_r - \mathrm{i}k_i)L/2)$, where $\mathrm{sinc}(x) = \sin(x)/x$. The limit of the usual spatial Fourier transform is strongly linked to the shift of poles in the lower half space, the amplitude of the peaks being reduced both by the amplitude of the mode $|\tilde{u}_n^m|$ and by k_i, these values being unknown. Similarly, the quality factor of the peak associated with the m-th mode can be low due to small L value and large k_i value. The influence of the length of the line over which the Laplace transform is performed clearly appears. This usually leads to problems in the determination of k_r^m when two modes are close one from the other or when they are highly (large value of k_i) attenuated. It is now proposed to show how the SLATCoW method allows to overcome these limitations.

The problem when trying to recover K^m is twofold : the amplitude and phase of the mode are unknown and the position of k_i^m in the complex s-plane is by definition unknown. Therefore, we will focus the analysis on the upper half space $s_i > 0$ where no mode K^m is included. Inspired by previous works on the recovery of the reflection coefficients of higher order modes propagating in a square cross-section impedance tube [4], the recovery of K^m is performed for each frequency by minizing the following cost function

$$F(|\tilde{u}_n^m)|, \phi^m, k_r^m, k_i^m) = \sum_{s_r} \sum_{s_i} \Big| U_n^{mes}(s) - L \sum_m |\tilde{u}_n^m| \exp(\mathrm{i}\phi^m) \times \exp((\mathrm{i}(k_r^m + \mathrm{i}k_i^m) - s_i - \mathrm{i}s_r)L/2) \frac{\sinh((\mathrm{i}(k_r^m + \mathrm{i}k_i^m) - s_i - \mathrm{i}s_r)L/2)}{(\mathrm{i}(k_r^m + \mathrm{i}k_i^m) - s_i - \mathrm{i}s_r)L/2} \Big|, \tag{3}$$

where $U_n^{mes}(s)$ is the spatial Laplace transform of the measured normal displacement, and $|\tilde{u}_n^m|$ and ϕ^m are respectively the theoretical amplitude and phase of the m-th mode. Note that the $\mathcal{L}_1$-norm is used in Eq. 3 because it leads to quite similar results as the usual $\mathcal{L}_2$-norm. The latter should be preferred to analyze measures with low signal- to-noise ratio. The minimization is performed under constrains with the Matlab®function *fminsearchbnd*. The number of modes which has to be recovered is determined a priori by looking at the usual spatial Fourier transform for each frequency. The recovery of both the amplitude and phase of each mode is more suitable than the direct determination of their real and imaginary parts. This particularly allows to avoid problems related to the unknown amplitude, overlapping of modes and to polarization of the mode. Roughly speaking, the recovery procedure could be performed in two steps : (i) the positions of the peaks at $s_i = 0$ give the values of k_r^m, (ii) at $s_r = k_r^m$ the exponential decay is related to the value of k_i^m. However, each mode strongly interacts in practice in the complex s-plane, which emphasizes the requirement of using a model involving all possible modes at a given frequency and the minimization of the previous cost function to efficiently determine all the possible K^m.

3 Application to guided elastic waves in porous material

Porous materials are known to be highly dissipative both in acoustics thanks to viscothermal losses and in mechanics through the solid-fluid phase interaction and the solid phase viscoelasticity. Seminal works by Boeckx *et al.*[1, 2] have paved the way for the characterization of the mechanical parameters that are intrinsic to the solid porous material phase by using guided waves. Nevertheless, these works mainly focus on the experimental recovery of the phase velocity $v_\phi^m = \omega/k_r$, due to a lack of both experimental data and analyzing tools for the efficient determination of the attenuation k_i^m. The present method is applied to experimentally determine both real and imaginary parts of the complex mode, therefore filling the gap for the attenuation measurement and enabling future works for characterizing viscoelastic parameters of porous material.

The experimental set-up, which is similar to the one used in[1, 2], is depicted in Fig. 1 (a).

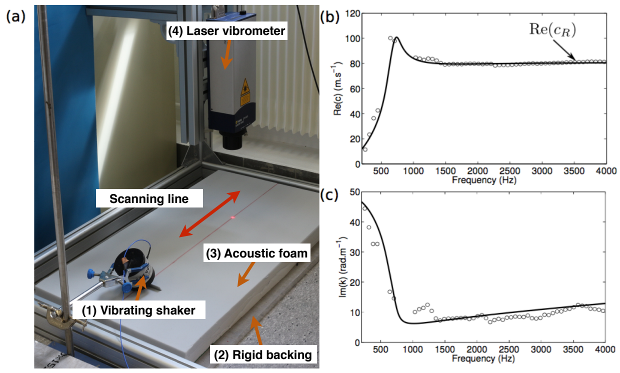

FIGURE 1 – (a) Experimental setup : (1) shaker Bruel and Kjaer type 4810, (2) rigid backing, (3) acoustic foam, and (4) laser vibrometer Polytec OFV-503, (b) real part of the velocity of the quasi-A_0 mode, and (c) attenuation ($\mathrm{Im}\,(k)$) of the quasi-A_0 mode.

A high porosity ($\phi > 0.95$) melamine foam sample of 85 cm length, 45 cm wide and 5.5 cm thick is glued on a rigid backing. The excitation is provided by a shaker (Bruel and Kjaer type 4810) which is rigidly attached to the sample with an adaptive part of “T” shape. This adaptive part is made of a threaded steel rod (20 mm length and 5 mm diameter) fixed to the shaker on one side and glued on a 1 mm thick aluminum plate of width 10 cm and height 1.5 cm on the other side. This plate is cut at the width edge (opposite to the threaded steel rod) and glued to the porous sample creating a line source at 15 cm from on edge of the sample. The plane wave excitation was experimentally verified and the resonance of this adaptive part was measured at 4500 Hz. Whilst the measurement could be ran at higher frequencies (up to 8000 Hz with a relatively good signal to noise ratio), results are only shown for frequencies below this limit. The excitation is 300 sine function equally spaced between 200 Hz and 4095 Hz. The normal displacement $u_n(x)$ is acquired at 801 positions along $L = 40$ cm with a laser vibrometer (Polytec OFV-503) mounted on a one dimensional robot, which moves the laser along the x-axis after each frequency measurement is accomplished, and connected to a spectral analyzer (Stanford Research Systems SR785), which allows us to directly measure $u_n(x)$ in the frequency domain. Each measurements is average over 100 periods.

The parameters of the porous materials, density ρ, porosity ϕ, flow resistivity σ, viscous and thermal characteristic length Λ and Λ', shear modulus $N = 38$ kPa (real part) and Poisson ratio $\nu = 0.3$ were determined independently using standard methods [5] and are given in Table 1. The so-called damping parameter Im(N), which is usually considered as a constant has been determine by fitting the experimental curves with the model. The present

Tableau 1 – Material parameters for the melamine foam.

ϕ	ρ(kg.m^{-3})	α_∞	σ(N.s.m^{-4})	Λ(μm)	Λ'(μm)
0.989	6.1	1	8060	215	215

method is applied one frequency after the other to obtained dispersion curves. More than 8 quasi-Lamb (because the sample is rigidly backed) modes can be recovered over this frequency range. This measurement is known to be difficult because of the highly dissipative nature of the sample, the low frequency excitation, and the large distance over which measurements have to be conducted. Indeed, for the resolution to be sufficient to discriminate all these modes, measurements should be performed over at least a few meters which is usually impossible. We choose to focus on the quasi-A_0 mode which was the most excited one. Problems arise because several modes overlap in the vicinity of this mode. The procedure was applied to recover 3 modes near the quasi-A_0 mode in order to remove the remaining components of the others. The real part of velocity (Re(c) = Re(ω/K) and the attenuation (k_i) versus frequency are plotted in Fig.1 (b) and (c) respectively, and compared to theoretical predictions obtained using Stroh formalism[6, 7] and a Müller algorithm to determine the complex roots of the corresponding complex dispersion relation. Measurement agree well with the theoretical predictions when the imaginary of N is fixed such that $N = 38 - 4$i kPa. This value is in accordance with the literature. Some obviously wrong measurements points were removed at low frequency. These results prove the efficiency of the present method to discriminate mode when several are overlapping. Furthermore, it was found that the Poisson ratio has a poor influence on the results, while the real part of N strongly influence the real part of the velocity and its imaginary part strongly influence the attenuation of the mode (which has enabled us to determine its value). The attenuation of the mode in porous materials are experimentally determined in a precise way thanks to SAW measurement for the first time. This pave the way to the experimental determination of the viscoelastic parameters of porous materials together with the development of refined models to account for viscoelasticity.

4 Conclusion

A new method for the recovery of the real and imaginary parts of wavenumbers from guided wave measurements is presented. This method, named SLATCoW (Spatial Laplace Transform for COmplex Wavenumber recovery), is based on a spatial Laplace transform of the measured displacement (or velocity) in the frequency domain, instead of the usual spatial Fourier Transform. The Laplace transform, providing informations both on the real and imaginary parts of the poles, is analyzed thanks to the minimization of a correctly chosen cost function (either $\mathcal{L}_1$-norm or $\mathcal{L}_2$ norm). This allows to reconstruct complex wavenumber (as well as the complex amplitude) of the modes, even when they are interacting one with the other. The SLATCoW method was applied to guided waves in a porous materials (attenuant in essence) in the kHz regime for the characterization of the viscoelastic properties the solid phase. The SLATCoW method was permitted to reconstruct the complex wavenumber of the quasi-A_0 mode. Besides the recovery of complex wavenumbers from guided wave measurements, the SLATCoW method paves the way to the experimental determination of the viscoelastic parameters of porous materials together with the development of refined models to account for viscoelasticity.

Acknowledgement

J.P. Groby gratefully acknowledge the support from the ANR Project METAUDIBLE No. ANR-13-BS09-0003 co-founded by ANR and FRAE. J.P. Groby and A. Duclos are grateful for the support of teh LMAc projects DECIMAP and PAVNat.

Références

[1] L. Boeckx, P. Leclaire, P. Khurana, C. Glorieux, W. Lauriks, and J. F. Allard, Investigation of the phase velocities of guided acoustics waves in soft porous layers, *JASA*, 1**17**, 2005

[2] L. Boeckx, P. Leclaire, P. Khurana, C. Glorieux, W. Lauriks, and J. F. Allard, Guided elastic waves in porous materials saturated by air under lamb conditions, *JAP*, 2005

[3] F. Wooten, Chapter 6 - dispersion relations and sum rules, in Optical Properties of Solids, edited by F. Wooten, 173-185 (Academic Press) (1972)

[4] J. Prisutova, K. Horoshenkov, J. -P. Groby, and B. Brouard, A method to determine the acoustic reflection and absorption coefficients of porous media by using modal dispersion in a waveguide, *JASA*, 136, 2947-2958 (2014)

[5] J. F. Allard and N. Atalla, Propagation of Sound in Porous Media : Modelling Sound Absorbing Materials 2e (Wiley and Sons, Chichester) (2009)

[6] G. Gautier, L. Kelders, J. -P. Groby, O. Dazel, L. De Ryck, and P. Leclaire, Propagation of acoustic waves in a one-dimensional macroscopically inhomogeneous poroelastic material, *JASA* 130, 1390-1398 (2011)

[7] O. Dazel, J. -P. Groby, B. Brouard, and C. Potel, A stable method to model the acoustic response of multilayered structures, *JAP*, 113, 083506 (2013)

[8] C. Prada, D. Clorennec, and D. Royer, Power law decay of zero group velocity lamb modes, *Wave Motion* 45, 723-728 (2008)

[9] G. Rix, C. Lai, and A. Spang Jr., In situ measurement of damping ratio using surface waves, *J. Geotech. Geoenviron. Eng.* ; 126, 472-480 (2000)

[10] G. Rix, C. Lai, and S. Foti, Simultaneous measurement of surface wave dispersion and attenuation curves, *Geotech. Test. J.*, 24, 350-358 (2001)

[11] S. Foti, Small-strain stiffness and damping ratio of pisa clay from surface wave tests, *Geotechnique*, 53, 455-461 (2003)

[12] A. S. Misbah and C. L. Strobbia, Joint estimation of modal attenuation and velocity from multichannel surface wave data, *Geophysics*, 79, EN25 ?EN38 (2014)