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Design of cubic stiffness for the absorber of Nonlinear Energy Sink (NES)

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The reduction of vibrations is a thematic evolution research particularly since the emergence of innovative absorbers Nonlinear Energy Sink (NES). This type of absorber is characterized by a secondary mass highly coupled via a non-linear stiffness to the main structure that needs to be protected. This nonlinearity allows an irreversible energy transfer from the main structure to secondary mass. The mastery of the nonlinearity is a key element for obtaining optimum performance. However, in practice it is difficult to obtain a cubic stiffness without linear part. In this article, a novel NES design leading to the award of a cubic stiffness is presented. For this, conical springs have been specifically sized to provide nonlinearity. To eliminate the linear term, the concept of negative stiffness is implemented from two cylindrical compression springs. The system was designed and sized. To validate the concept, an analytical study based on the method of multiple scales is presented. Future developments will aim manufacturing and experimental validation of the prototype.

1 Introduction

Mitigation of unwanted vibration is an important issue in many fields of engineering. Since the emergence of innovative absorber Nonlinear Energy Sink (NES), more attentions were paid to this promising technique [1]. This type of absorber is characterized by a secondary mass highly coupled with a non-linear stiffness to a main structure that needs to be protected. By trigging resonances between related nonlinear normal modes, the nonlinearity allows an irreversible energy transfer from the main structure to secondary mass [2]. Unlike the traditional linear absorber Tuned Mass Damper (TMD) that needs to be tuned to a specific natural frequency, NES can passively absorb the energy over a wide range of natural frequencies [2-4]. Additional with a relatively small mass, make it particularly attractive in a wide variety of applications such as space and aero-structure, vibrating machinery, building and vehicle suspensions [4,5].

The mastery of the nonlinearity is a key element for obtaining optimum performance. Depending on the type of nonlinearity, different kinds of NES have been proposed: oscillating dissipative with pure cubic stiffness [6,7], piecewise stiffness [8,9], rotational elements [10] and sinks undergoing vibro-impacts [11,12]. As far as the pure cubic NES, it has been shown that this configuration is most effective at moderate–energy regimes. Yet in practice it is difficult to obtain a cubic stiffness without linear part. In our recent approaches, the essential cubic stiffness was mostly realized by adopting the construction of two springs with no pretension [7]. Due to the self-geometric nonlinearity, the springs stretch in tension thus creating the cubic force. However, this classical type can't effectively profit spring's compression and tension performance, resulting in a large size vertical structure attached to the main system; Addition of a relatively weak nonlinear stiffness existing at the beginning extension, leads to the whole cubic term approximated to a linear term. Therefore, how to implement cubic stiffness elements practically is still an important issue to broaden the application of NES.

In this article, a novel NES design leading to the award of a strongly cubic stiffness is presented. The structure is as follows: section 2 is devoted to conception of conical springs, which is specifically sized to provide nonlinearity; in section 3, a negative stiffness mechanism is implemented from two cylindrical compression springs to eliminate the linear term; in the next section, to validate the concept an analytical study based on the method of multiple scales is presented; Finally, concluding remarks and future developments are addressed.

2 Conical spring design

Owing to the self-nonlinearity, conical spring possesses the advantage of providing variable spring rates and varying natural frequencies, additional it can avoid buckling at large deflections. For this, two conical springs with a constant pitch and a constant coil diameter are adopted. Considering the strong nonlinearity and lower installation height, the shape of telescoping spring is used, as shown in Fig.1.

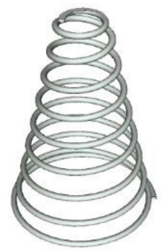

Fig.1 Telescoping conical spring

The dynamical behavior of conical spring with a constant pitch can be classified as linear and nonlinear part. To distinguish the two phases, three particular points are introduced, as shown in Fig.2: Point O corresponds to the spring free state; Point T means the transition point that starts the nonlinear behavior; Point C represents the state of maximal compression.

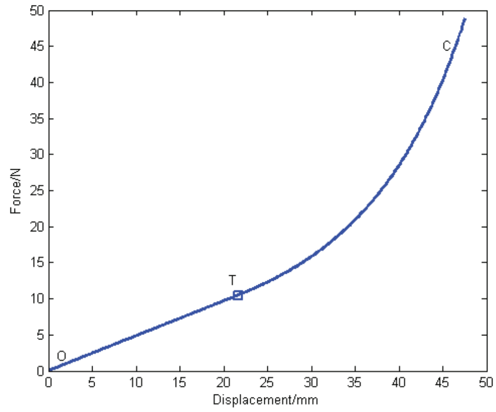

Fig.2 Conical spring characteristic

In the linear phase (from point O to T), the largest coil is free to deflect as the other coils, so the load-deflection relation is linear and the stiffness can be expressed as [13]:

$$R = \frac{Gd^4}{2n_a(D_1^2 + D_2^2)(D_1 + D_2)} \tag{1}$$

In the nonlinear regime (from point T to C), the active coils are gradually compressed to the ground. During this regime, n_f coils are free, and $n_a - n_f$ coils are compressed to the ground, which means that these coils have reached their maximum physical deflection. By compressing progressively, n_f decreases from n_a to 0 and leads to a gradual increase of the spring stiffness. The load-deflection relation is shown as follows [13]:

$$\Delta(\mathrm{P}) = \frac{2PD_1^4 n_a}{Gd^4(D_2 - D_1)}[(1+(\frac{D_2}{D_1}-1)\frac{n_f}{n_a})^4 - 1] + (L_a - L_s)(1-\frac{n_f}{n_a}) \tag{2}$$

To profit the nonlinear performance of conical spring, the connecting type of spring is proposed in Fig.3.

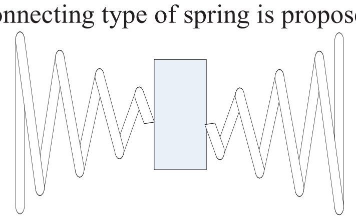

Fig.3 Connecting type

However, this configuration exist the problem of possessing piecewise stiffness of linear and nonlinear part. To skip the linear phase, a method of pre-compressing at transition point is adopted. By changing the initial original point, the behaviors of two conical springs can respectively belong to linear and nonlinear regimes simultaneously. Supposing the right direction of vibrating as positive, setting the left spring as first one and the right as second one, we can obtain the new load-deflection relation, as shown in Fig.4. By compressing progressively, the force of second spring increases nonlinearly, while the first decreases linearly. When the displacement reaches the value of transition point's deflection, it returns back to the free length and starts to work at tension regime. Combining the two spring's curves, the composed stiffness curve is obtained and it is obviously observed that the new curve is smooth and no longer piecewise as before.

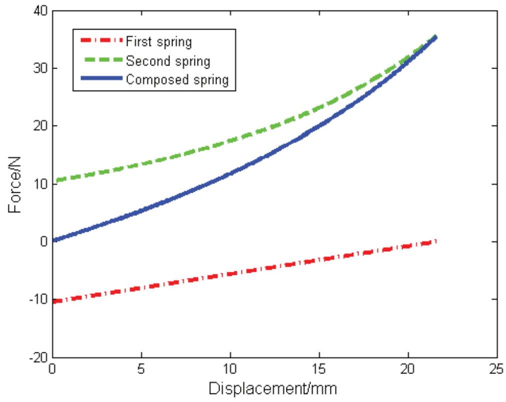

Fig.4 Pre-compressed characteristics

To analysis the internal polynomial components, the method of polynomial fitting is used to obtain the new load-deflection relation, as follows:

$$P = a_1 x + a_2 x^2 + a_3 x^3 \tag{3}$$

In this polynomial, the linear term $a_1 x$ is hardly to be eliminated owing to the superposition of linear and nonlinear part, while the square term $a_2 x^2$ is possible to make its value small.

After optimizing the parameters of conical spring (the mean diameter D_1 and D_2), the new polynomial components is obtained and presented in Fig.5. It can be observed that the curve of cubic and linear term is almost closed to the original one, by means that the contribution of square term is small that can be almost neglected.

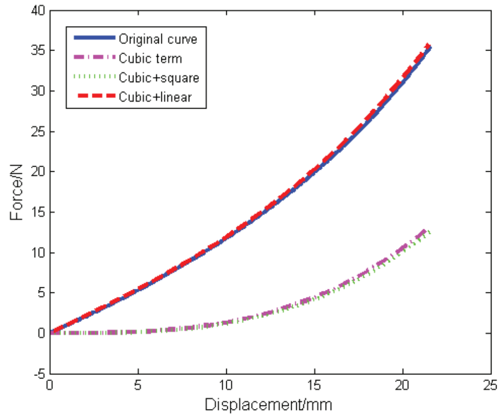

Fig. 5 Polynomial components

3 Negative stiffness mechanism

To eliminate the proposed linear term, adding a new term which has the negative stiffness in the translational direction seems be a way forward. For this, a negative stiffness mechanism is implemented from two cylindrical compression springs, and the structure is shown in Fig.6.

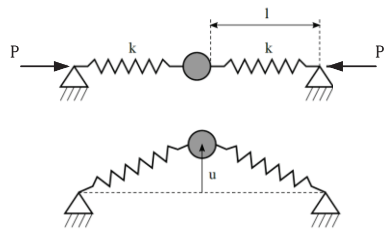

Fig.6 Negative stiffness mechanism

Based on Taylor expansion, the force-displacement relationship of pre-compressing at the length of l_p is expressed as:

$$f = 2\frac{P}{l}u - \frac{kl+P}{l^3}\cdot u^3 \tag{4}$$

Superposing the force with the one of conical spring in the translational direction, the composed force can be depicted as:

$$P_m = (a_1 - 2k\frac{l_p}{l})\cdot x + (a_3 + k\frac{l_p + l}{l^3})\cdot x^3 \tag{5}$$

As can be seen from Eq. (5), if we set $a_1 = 2kl_p / l$, the equation will be left with the pure cubic term, and the coefficient of cubic term will increase a little larger.

Based on the proposed methods, a small sized NES system providing strongly nonlinear stiffness is designed, and the assembly drawing is presented in Fig.6, of which the component parts are spherical plain bearing, linear guide, conical spring, linear spring and NES mass.

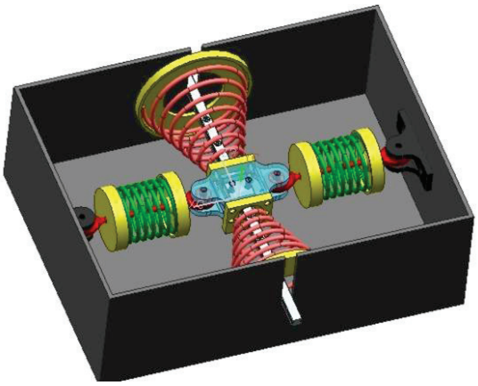

Fig.6 Cubic stiffness absorber system

To make certain the conical springs work in the compression state, the maximum displacement of NES is limited at the deflection of transition point. The corresponded characteristic curve is presented in Fig.7. It shows that the stiffness in the required working range is pure cubic and strongly nonlinear.

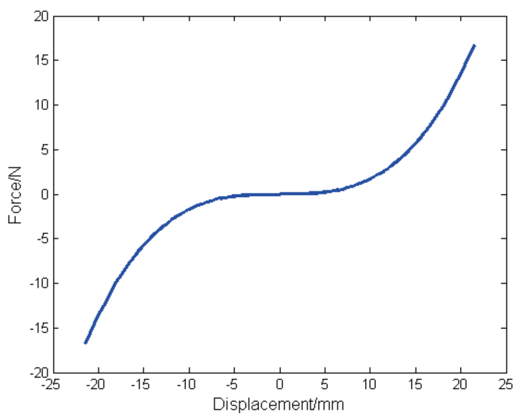

Fig.7 Pure cubic stiffness curve

4 Analytical study

To validate the concept, an analytical study of a harmonically excited linear oscillator (LO) strongly coupled to a NES is presented, as shown in Fig. 8.

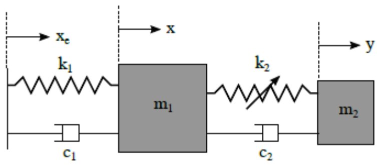

Fig.8 LO coupled with NES

The governing equations of motion of this system are given by:

$$m_1\frac{d^2x}{dt^2}+c_1\frac{dx}{dt}+k_1x+c_2\left(\frac{dx}{dt}-\frac{dy}{dt}\right)+k_2(x-y)^3=k_1x_e+c_1\frac{dx_e}{dt}$$
$$m_2\frac{d^2y}{dt^2}+c_2\left(\frac{dy}{dt}-\frac{dx}{dt}\right)+k_2(y-x)^3=0 \qquad (6)$$

The imposed harmonic displacement x_e is expressed as:

$$x_e=G\sin\tilde{\Omega}t \qquad (7)$$

By scaling parameter such as $\tau=\omega_0 t$, $m_2/m_1=\varepsilon$, the transferred equations are obtained:

$$\begin{aligned}&\ddot{x}+x+\varepsilon\lambda_1\dot{x}+\varepsilon\lambda_2(\dot{x}-\dot{y})+\varepsilon K(x-y)^3\\&=\varepsilon F\cos\Omega t-\varepsilon^2\lambda_1F\sin\Omega t\\&\varepsilon\ddot{y}+\varepsilon\lambda_2(\dot{y}-\dot{x})+\varepsilon K(y-x)^3=0\end{aligned} \qquad (8)$$

Where the dots denote differentiation with respect to τ and the following parameters are defined:

$$\{\omega_1,\omega_2,\lambda_1,\lambda_2,\Omega,\mathrm{K},\mathrm{F}\}=\left\{\sqrt{\frac{k_1}{m_1}},\sqrt{\frac{k_2}{m_2}},\frac{c_1}{m_2\omega_1},\frac{c_2}{m_2\omega_1},\frac{\tilde{\Omega}}{\omega_1},\frac{\omega_2^2}{\omega_1^2},\frac{G}{\varepsilon}\right\}$$

New variables are introduced as follows:

$$v=x+\varepsilon y,\quad w=x-y \qquad (9)$$

To study the response in the vicinity of the 1:1resonance, the following complex variables are introduced:

$$\phi_1e^{i\Omega t}=\dot{v}+i\Omega v,\quad \phi_2e^{i\Omega t}=\dot{w}+i\Omega w \qquad (10)$$

Substituting Eq. (9) and Eq. (10) into Eq. (8) and keeping only the secular term containing $e^{I\Omega t}$ yields the following slow modulated system:

$$\begin{aligned}&\dot{\phi}_1+\frac{i\Omega}{2}\phi_1+\frac{\varepsilon\lambda_1}{2(1+\varepsilon)}(\phi_1+\varepsilon\phi_2)\\&-\frac{i(\phi_1+\varepsilon\phi_2)}{2\Omega(1+\varepsilon)}-\frac{\varepsilon F}{2}-\frac{i\varepsilon^2\lambda_1F\Omega}{2}=0\\&\dot{\phi}_2+\frac{i\Omega}{2}\phi_2+\frac{\varepsilon\lambda_1}{2(1+\varepsilon)}(\phi_1+\varepsilon\phi_2)-\frac{i(\phi_1+\varepsilon\phi_2)}{2\Omega(1+\varepsilon)}\\&+\frac{\lambda_2(1+\varepsilon)}{2}\phi_2-\frac{3iK(1+\varepsilon)}{8\Omega^3}\phi_2^2|\phi_2|-\frac{\varepsilon F}{2}-\frac{i\varepsilon^2\lambda_1F\Omega}{2}=0\end{aligned} \qquad (11)$$

Considering the small parameter ε, the method of multiple scales is introduced in the following form:

$$\phi_i=\phi_i(\tau_0,\tau_1,\ldots),\ \frac{d}{d\tau}=\frac{\partial}{\partial\tau_0}+\varepsilon\frac{\partial}{\partial\tau_1}+\ldots,$$
$$\tau^k=\varepsilon^k\tau,\ k=0,1,\ldots \qquad (12)$$

Introducing Eq. (12) into Eq. (11) and equating coefficient in order ε^0 and ε^1:

$$\begin{aligned}&\varepsilon^0:\frac{\partial}{\partial\tau_0}\phi_1=0\\&\frac{\partial}{\partial\tau_0}\phi_2+\frac{\lambda_2}{2}\phi_2+\frac{i}{2}(\phi_2-\phi_1)-\frac{3iK}{8}\phi_2^2|\phi_2|=0\end{aligned} \qquad (13)$$

$$\begin{aligned}&\varepsilon^1:\frac{\partial}{\partial\tau_1}\phi_1+\frac{i}{2}(\phi_1-\phi_2)+i\sigma\phi_1+\frac{\lambda_1}{2}\phi_1-\frac{F}{2}=0\\&\frac{\partial}{\partial\tau_1}\phi_2+\frac{i}{2}(\phi_1-\phi_2)+\frac{i\sigma}{2}(\phi_1+\phi_2)+\frac{\lambda_1}{2}\phi_1\\&+\frac{\lambda_2}{2}\phi_2-\frac{3iK(1-3\sigma)}{8}\phi_2^2|\phi_2|-\frac{F}{2}=0\end{aligned} \qquad (14)$$

Substituting $\phi(\tau_1)=N_2e^{i\delta_2}$ into Eq. (13), the equation of slow invariant manifold (SIM) is presented as follows:

$$\begin{aligned}&|\phi_1|^2=\lambda_2^2Z_2+Z_2-\frac{3K}{2}Z_2^2+\frac{9K^2}{16}Z_2^3\\&Z_2(\tau_1)=N_2(\tau_1)^2\end{aligned} \qquad (15)$$

Introducing the parameters of previous cubic stiffness absorber of Tab.1, the structure of slow invariant manifold (SIM) is obtained, as shown in Fig.9. This topologic structure is mainly responsible for the possible occurrence energy pumping and it may give rise to the strongly modulated response (SMR) .The detailed description can refer to [14].

Tab.1 Parameters of NES

Physical parameters			
m_1	14*kg*	k_2	$1.68\times10^6\ N/m^3$
m_2	0.14*kg*	C_1	4*Ns/m*
k_1	$3\times10^4\ N/m$	C_2	0.4*Ns/m*
Reduced parameters			
ε	0.01	λ_2	0.062
λ_1	0.62	K	5620
The SMR threshold			
G_{1c}	0.045*mm*,	G_{2c}	0.16*mm*

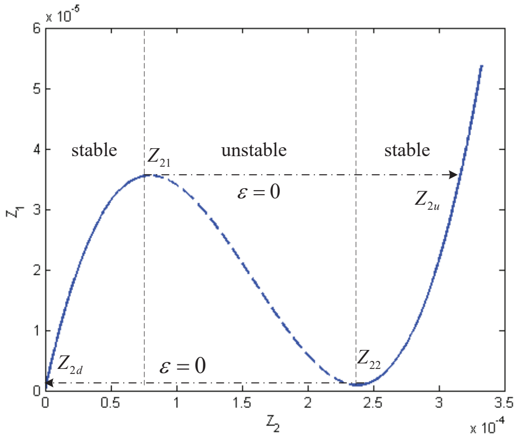

Fig.9 SIM structure

Based on the Eq. (8), setting the initial condition state as zero and the excitation amplitude as 0.1mm, the numerical responses of NES and LO are obtained in Fig.10. A quasi-periodic regime with a slow evolution of the amplitudes of both oscillators is observed. For LO, the amplitude increases and decreases repeatedly in a regular fashion. For the amplitude of NES, it can be classified into two levels: a small one corresponding to the growth of LO amplitude, a large one when LO amplitude decreases. This alternating regime of SMR proves the jump phenomenon of SIM (the hypothetic “jump” between the stable branches is noted by arrow), and it has been demonstrated that this regime is rather effective for vibration mitigation [15].

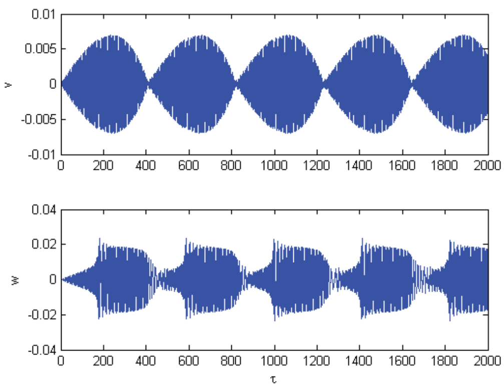

Fig.10 Strongly modulated response with G=0.1mm

As can be seen from Fig.10, the maximum amplitude of NES is near to 20mm, by means that this value is closed to the deflection of transition point and the performance of conical spring is well profited. Moreover, it can be observed that this system with strong nonlinearity could effetively abosrber and dissipate targeted energy uner a small excitation amplitude.

5 Conclusion

In this paper, a novel design NES of cubic stiffness without linear part is presented. For this, two conical springs were specifically sized to obtain the strong nonlinearity. To skip the linear phase of conical spring, a method of pre-compressing at the transition point was used, so to provide the polynomial components only with linear and cubic term. To eliminate the linear term, the concept of negative stiffness was implemented from two cylindrical compression springs. Based on the proposed methods, a small sized NES system providing strongly nonlinear stiffness was developed. To validate the concept, an analytical study based on the method of multiple scales was presented. The results showed that at specified excitation amplitude, this system could passively transfer the unwanted disturbance energy with the response of SMR. Moreover, owing to the strong nonlinearity, this type could effetively abosrber and dissipate targeted energy uner a small excitation, which makes it possible to broaden the NES application in vibration mitigation of fine mechanics. Further developments will aim manufacturing and experimental validation of this prototype.

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