Thirty years since “Diffuse Sound Reflection by Maximum-Length Sequences”: Where are we now?

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This year we are celebrating the thirtieth anniversary of Schroeder’s seminal paper on sound scattering from maximum length sequences. This paper, along with Schroeder’s subsequent publication on quadratic residue diffusers, broke new ground because they contained simple recipes for designing diffusers with known acoustic performance. So what has happened in the intervening years? As with most areas of engineering, room acoustic diffusers have been greatly influenced by the rise of digital computing technologies. Measurement technologies have greatly advanced enabling the measurement and characterising of diffusers in greater detail. Numerical methods have become much more powerful, and this has enabled predictions of surface scattering to greater accuracy and for larger scale surfaces than previously possible. These tools have enabled a great deal of knowledge and understanding to be amassed about the principles of diffuser design and application. Also, architecture has gone through a revolution where the forms of buildings have become more extreme and sculptural. Acoustic diffuser designs have had to keep pace with this to produce shapes and forms that are desirable to architects. To achieve this, design methodologies have moved away from Schroeder’s simple equations to brute force optimisation algorithms. This paper will look back at the past development of the modern diffuser, explaining how the principles of diffuser design have been devised and revised over the decades. The paper will also look at the present state-of-the art, and dream about the future. However, even now, there is much that can be learnt from simple mathematics and number theory.

1 Introduction

At the start of Schroeder’s seminal paper on diffuse sound reflection by maximum-length sequences, Schroeder posed the following question[1]:

“What wall shape has the highest possible sound diffusion in the sense that an incident wave from any direction is scattered evenly in all directions?”

In this short and pithy paper – it is less than 2 pages long – Schroeder proposes making diffusers based on Maximum Length Sequences (MLS) as one answer to this question. However, even MLS surfaces do not completely satisfy this definition of optimal sound diffusion, in the following three decades, much more has been learnt about diffusers, for instance that spatial distribution is not the only important characteristic of diffusers and that for large surfaces it is impossible to have broadband even scattering in all directions. This paper summarises some of what has been learnt in the last 30 years about diffuser design.

2 Bandwidth

The bandwidth of Schroeder diffusers have been most commonly defined in terms of the maximum well depth and the well width[2]. The maximum depth allowable for treatment is normally specified by non-acousticians, even though it has a crucial acoustic consequence. If the diffuser is too shallow compared to the wavelength, it doesn’t perterb the sound wave. An approximate lower limit is when the maximum depth is equal to a quarter of a wavelength. Some ingenious solutions to reduce this limitation have been proposed:

- Make some of the wells into Helmholtz [3,4,5] or membrane resonators [6] and so lower the frequency at which waves are perterbed; but these surfaces become less diffuse with increasing frequency, which is usually undesirable.
- Use active impedance technology [7] to virtually extend well depths; but these diffusers are rather expensive and difficult to implement.
- Use absorptive material to form hybrid surfaces [8]; but these are only useful when partial absorption is wanted.
- Bend or fold wells to exploit the unused space in the body of the diffuser [9.10.11], which works well but is more expensive to make.

One low frequency limitation that is often overlooked is a limitation caused by repetition. Because diffusers are often used with many identical copies placed side by side (a periodic arrangement), the spatial scattering is dominated by grating (or spatial aliasing lobes). If the wavelength is large compared to the period width, then the diffuser only generates the zero order lobe pointing in the specular direction. Without the higher order lobes, no obliquely propagating energy can be achieved. This effect can be seen in the polar responses shown in Figure 1. This can be overcome by emoving repetition by using a single wide diffuser, or by using an appropriate diffuser arrangement or modulation scheme [12,13,14] - see below.

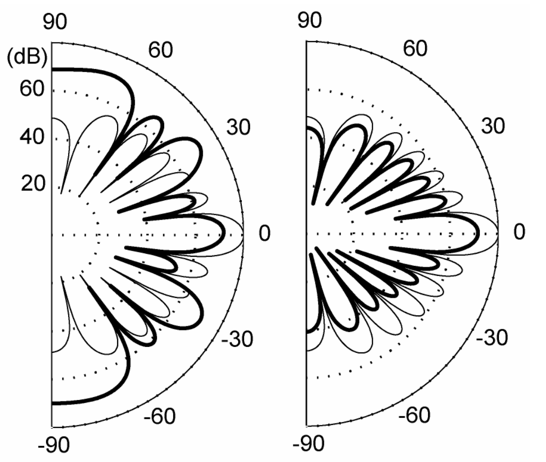

Figure 1. The pressure scattered from two Quadratic Residue Diffusers (thick lines) and a plane surface (both thin lines) at 1000Hz. The left figure's QRD has a period width 3x the period width of the right figure's QRD. (After Cox and D'Antonio [6])

In the simplest Schroeder diffuser design theory, there is an assumption of plane wave propagation in the wells. Consequently, the well width is often quoted as being the high frequency limit of the device. However, the diffuser still continues to scatter above this limit, just in a less controlled manner.

It is not possible to make the diffuser well widths very narrow to increase the high frequency validity of the design equations. If very narrow and deep wells are made, then the diffuser is likely to suffer from two defects. First, the viscous boundary layer losses at the well sides are likely to be significant leading to the diffuser being overly absorbent [6]. Second, when very narrow wells are employed, at low frequencies the scattering ability is compromised because there is insufficient path length difference between the scattered waves from neighbouring wells. An elegent solution to this is to use a fractal construction [15]. This is where small diffusers for high frequency scattering are combined with large diffusers for low frequency dispersion. An example of this type of diffuser is shown in Figure 2. Figure 3 shows how the fractal construction improves the diffusion against a more traditional Schroeder diffuser. An alternative solution is to use a curved surface, provided the curves are not too gentle, as these will not have problems with plane wave limits.

Figure 2. A fractal schroeder diffuser (After D'Antonio and Konnert [15]).

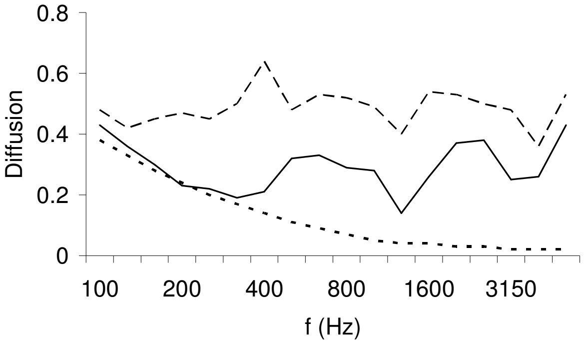

Figure 3. Diffusion for: ——— an N=7 QRD; – – – – a fractal QRD, and - - - - - -a plane surface (Data from Cox and D'Antonio[6]).

3 Critical and discrete frequencies

In Schroeder's original paper, he noted that maximum length sequences operate only over about an octave. The reason for this is that an octave above the critical frequency, all wells of a MLS diffuser have a reflection coefficient of 1, and consequently the surface reflects like a plane surface. The issue of critical frequencies appears to have received little attention until revisited by Angus [16], who gave solutions to the problem. The essential source of the problem is that number-theoretic diffusers are based on integer number sequences, and so the depths are related to each other by integers, and consequently, there will be a set of frequencies, which are multiples of the design frequency, when the reflection coefficients will all be 1. A more recent study [17] has shown that the problem of critical frequencies extends above the plane-wave high frequency limit of the diffusers. Figure 4 shows evidence of a critical frequency for an N=7 QRD at ≈3.5kHz. A simple solution to the problem is to use a large prime number generator for the diffusers to place the first critical frequency outside the design bandwidth. However, diffusers with small prime numbers are more popular because they are simpler and cheaper to make.

One solution is to use non-integer based sequences [18] so the depths are not integer related. Alternatively [19], if a computer is tasked to find the best diffuser depths, using a numerical optimisation algorithm, then the issue of critical frequencies does not arise. Figure 4 also shows the diffusion for an optimised diffuser showing better diffusion. Alternatively, a diffuser profile, such as an optimised curved surface, which does not have a series of wells will not suffer from critical frequencies.

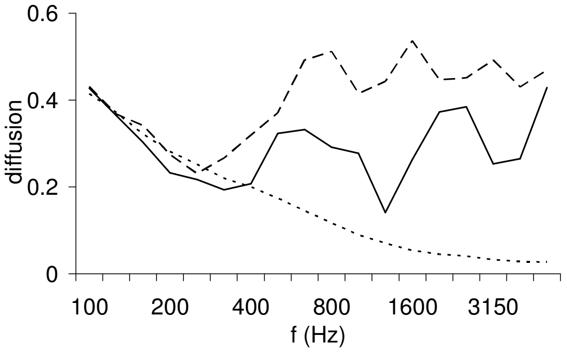

Figure 4. Diffusion for: ______ an N=7 QRD; ------ an optimised diffuser, and ....a plane surface (After Cox and D'Antonio [17]).

The use of optimisation also overcomes a problem that has received little attention in the literature. Strictly speaking, a number theoretic diffuser only works for a set of discrete frequencies which are multiples of the design frequency. This is not the same as having diffusion which covers a design bandwidth, although Schroeder diffusers are complex surfaces, and they do provide reasonable scattering between these discrete frequencies. This is more problematic for primitive root diffusers, which are supposed to form a diffuser suppressing scattering in the specular direction, because this suppression only happens at a few discrete frequencies across the design bandwidth [20].

4 Absorption

The absorption from Schroeder diffusers has received some attention over the years. Schroeder diffusers primarily absorb because of high energy flows from wells in resonance to wells out of resonance, and ¼ wave resonant absorption in the wells, especially if the wells are rather narrow. Figure 5 gives some typical absorption coefficients for commercial diffusers.

There are two key aspects to achieving low absorption from these surface, first it is important not to cover the diffusers. Around the well entrance the particle velocity is high, and so placing resistive material near the well entrance will result in excess absorption as Figure 5 shows. It is also important to ensure the diffuser is well made as poor construction, such as slits in the well bottoms opening to cavities behind can cause surprisingly large amounts of absorption.

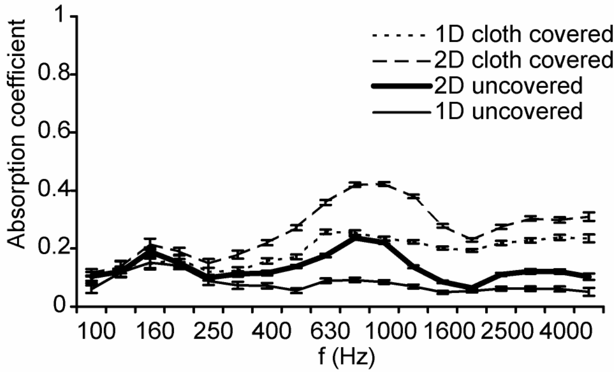

Figure 5. Random incidence absorption coefficients measured using ISO-140 for various Schroeder diffusers (After Cox and D'Antonio [6]).

Indeed, several researchers have investigated how these structures can be exploited to make absorbers [4,5,10]. While this research has shown that high absorption can be achieved (close to 1 over a reasonable bandwidth), the cost of making these structures currently makes them difficult to exploit commercially. They do have some advantages over mineral wool in being more robust and washable.

Using other shapes of diffusers has the advantage of reducing the risk of excess absorption in applications where absorption needs to be minimised.

5 Measurement of diffusion

In Schroeder's original paper, he examined the quality of the diffusion by examining the energy of the grating lobes. This is ultimately not a useful measure because diffusers do not have to be periodic and so do not necessarily have grating lobes. Consequently, there has been considerable effort to devise a parameter that can characterise the quality of diffuse reflection for use in design work and for surface specification. In 2001 this work produced an Audio Engineering Society Standard Information Document [21] which details a free field method for measuring diffuser quality in a diffusion coefficient. The scattered polar response from a surface is measured, and then the polar response characterised by a single figure of merit (which is based on the autocorrelation function). The method is now being considered as a working item for ISO WG25.

Alongside this work, a need for a coefficient to characterise the scattering from surfaces for computer models has been demonstrated. To take an example, in the first round robin study of room acoustic models [22], three prediction models were found to perform significantly better than others. These three prediction models produced results approximately within one subjective difference limen, while the less successful computer models produced predictions inaccurate by many difference limen. What differentiated the three best models from the others was the inclusion of a method to model surface scattering. There are many different methods for incorporating diffuse reflections into a geometric room acoustic model [23], and all require a scattering coefficient to measure the fraction of the reflected energy scattered into non-specular direction. A method to measure a scattering coefficient has now been incorporated into an ISO standard [24].

While this scattering coefficient might satisfy the need for modellers to improve the accuracy of geometric models, it is unlikely that it is sufficiently nuanced to properly measure the quality of diffusers.

6 Grating lobes

The original designs by Schroeder produce a series of grating lobes in their polar response due to repetition. This is not a problem when there is a large number of grating lobes present, because when a typical analysis bandwidth, such as one-third octave bands are considered, then these lobes will tend to average out to a roughly even scattered polar response. However, the typical diffuser sizes which are convenient to build and manufacturer, mean that diffusers often only have a sparse number of lobes for crucial frequency ranges. Figure 6 shows the number of lobes present as a function of the period width. If we assume that we want at least 7 lobes, than a diffuser with a 0.3m period width is only going to achieve this above 4kHz.

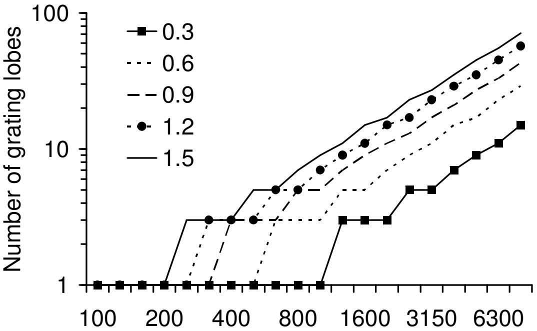

Figure 6. The number of grating lobes as a function of period width of a diffuser as shown in the legend in metres.

The solution of making a single period of a large Schroeder diffuser is often impractical. It can be expensive to achieve, and also the random appearance it generates is often not wanted by interior designers. A neat solution to the problem of sparse grating lobes is to use a modulation scheme. There are a few possible schemes [14]. An asymmetric narrow diffuser, say an N=7 primitive root diffuser, can be placed on the wall in a random order. Some of the diffusers are flipped in the arrangement so that the diffuser is no longer periodic. If the base diffuser depths are based on: {1,3,2,6,4,5}, then the flipped sequence has depths based on {5,4,6,2,3,1} and then 4 periods of this device modulated by a sequence {1,1,0,1} would be:

{1,3,2,6,4,5, 1,3,2,6,4,5, 5,4,6,2,3,1, 1,3,2,6,4,5}

The question then arises what is the best arrangement of the diffusers, which is the best diffusers in the arrangement to flip? The solution again lies in number theory, where a large number of binary sequences have been developed (the most famous being MLS) with optimal autocorrelation properties [18].

Even with curved diffusers, the general principle of modulation can be applied as Figure 7 shows. This is a good example of learning general principles from number theory.

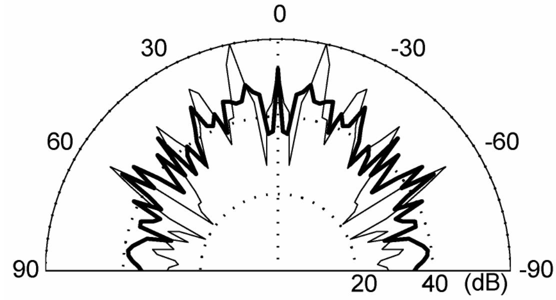

Figure 7. The scattered polar response from a —— periodic and —— modulated arrangement of curved diffusers. (After Cox and D'Antonio [6]).

7 Appearance

Since the invention of Schroeder diffusers, there have been many changes in architectural style. Indeed, it is a testiment to the success of the design philosophy of Schroeder diffusers, that these have continued to be specified and used for so many decades. However, modern architecture has moved on from simple rectilinear geometries, especially in prestige buildings (like the Guggenheim museum, Bilbao shown in Figure 8). Architects now excercise greater freedom in choosing the form of the buildings, and so it is important that acoustic treatments are sympathetic with a buildings design. Unfortunately, to fit with modern styles, it is not possible to use Schroeder's ingenious phase-change surface or the design equations developed. It is necessary to find a new design philosophy that can deal with arbitrary forms; such a design philosophy is numerical optimisation [25].

Figure 8 The Guggenheim Museum in Bilbao

In numerical optimisation, a computer is tasked with finding the shape with the best acoustic performance while being constrained to meet the architectural look required. For instance, Figure 9 shows an example of an optimised curved diffuser which is designed so it can be tiled in any orientation, enabling the architect to decide whether they want a periodic or random arrangement. Learning from number theory and modulation, we know that a random arrangement would be best at dispersion, but now it is possble for the architect to decide the trade off between acoustic performance and appearance in collaboration with the acoustic consultant.

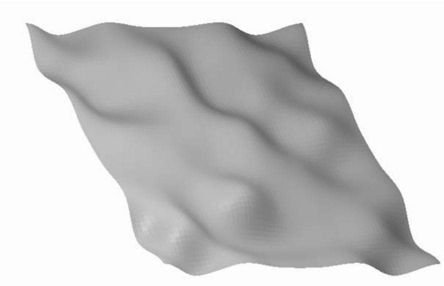

Figure 9. An optimised curved diffuser

8 Hybrid surfaces

Number theory can also be applied to absorption technology. For many decades, people have been arranging absorbers in studio spaces in a random fashion, to promote dispersion from the reflected waves. Building on the principle of Schroeder diffusers, Angus [8], showed that rather than using a random arrangement, it was best to use a pseudo-random arrangement based on a number theoretic sequence.

Figure 10 shows a Binary Amplitude Diffuser (BAD panel) which is formed by taking a 1023 length MLS and folding into a 31x33 array using the Chinese Remainder theorem [26]. The white holes allow the sound through to a backing layer of mineral wool, whereas the dark areas are reflecting. At mid-high frequencies, these surfaces cause partial absorption, with the reflected sound being diffused.

A simple mathematical analysis of these surfaces shows that the diffusion is limited by the large amount of flat reflected area. The surface only offers the chance of unipolar reflection coefficients, R=0 or 1, and so it is not possible to create waves of opposite phase to cancel the specular reflection. As evident from number theory, better scattering would be obtained from bipolar reflection coefficients, R=-1 or +1. Alternatively, by bending the panel [27], it is possible to further spatially disperse the reflection.

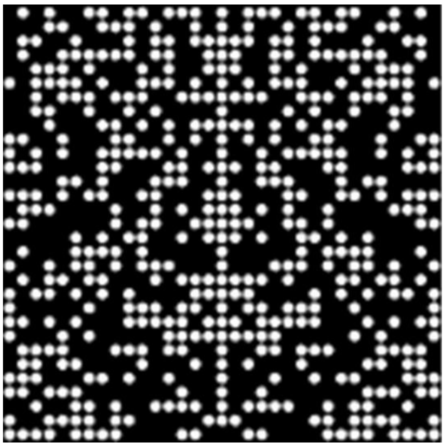

Figure 10. A hybrid diffusers using a folded MLS sequence.

9 Conclusions

Much has been learnt since the invention of diffusers based on number theoretic sequences. The original number theoretic diffusers offered defined acoustic scattering from simple design equations. Now it is possible to design diffusers which have shapes more in keeping with modern architectural forms, and these diffusers have better acoustic perforamce. Methods for predicting, measuring and evaluating the scattering from diffusers have been developed and improved. The absorption of diffusers have been measured and the mechanisms researched and understood.

How diffusers should be grouped is now better understood. The role that repetition plays in the sound dispersion has lead to the development of modulation schemes to improve scattering. However, most of our understanding about diffuser application, about where it is appropriate to use them, comes from prescendence rather than systematic scientific study. The subjective importance of diffusers has been investigated but not in sufficient detail, and so this is an important area for future research.

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