An Adaptive Optimization Approach to Active Cancellation of Repeated Transient Vibration Disturbances

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Vibration control of lightly damped structures such as space-borne optical platforms can be difficult when the disturbances are broadband transients. Such transients can be produced by the action of latches or positioning devices such as thrusters, inertia wheels and controlled moment gyroscopes. In some cases it may not be possible to use the system until the vibrations have decayed. Active vibration control, which relies on knowledge of structural transfer functions, disturbance waveforms, and spatial location, can be used to “block” the disturbance. However, the broadband nature of the disturbance and the natural variations in structures due to the environment and aging make knowledge of the transfer functions uncertain. But the fact that the transient disturbance is broadband, and is the result of a command, gives us an opportunity to bring signal processing techniques originally developed for machinery diagnostics to bear on this problem.

The signal processing method of “cepstral windowing” allows us to substitute more robust and stable estimates of the needed transfer functions. Knowledge of the time and location of the disturbance allows us to present a correction signal to a strategically placed actuator that will oppose the disturbance force. Reduction can be further enhanced by “adaptive optimization” of the correction signal, whereupon the response of the structure to repeated disturbances is analyzed. This paper discusses the use of source waveform recovery techniques in combination with an optimization procedure to reduce the amplitude and duration of repeated transient disturbances. Example results of applying these methods to a truss structure are given.

1 Introduction

Vibratory disturbances to aerospace structures, and to deployable spacecraft in particular, arise from transient forces applied to the structure. The resultant long settling time may interfere with the timing and accuracy of positioning, as well as other functions. A spacecraft is generally a lightweight, stiff, and poorly damped structure. The forces on the structure are often “planned” since they are the result of actuators, thrusters, and other mechanisms used for alignment, positioning, and functioning of the spacecraft. Since they are planned and controlled, the timing and location of the excitation is known. That allows the application of an opposing excitation at the proper time and place if the waveform of excitation can be determined. Therefore, active control of the vibrations that result from these “known” disturbances is a viable prospect. However, most established methods are effective on only a handful of modes, and require precise knowledge of the transfer function (TF) between the canceling force and the vibration observation point. The TF can change as a function of temperature, loading and age, and it can also vary from one structure to another of the same design. These effects of environment and usage result in TFs that are variable and therefore uncertain.

Our methodology is based on reducing the sensitivity of the active cancellation to details of the TF, as well as on introducing an adaptive optimization technique to account for long term changes in either the TF or the disturbance forces. Cepstral processing offers a potential way to reduce the sensitivity to the details of the TF, as well as stabilize the structural TF for inverse filtering [1]. Used extensively in speech processing and to eliminate the effect of echo from received signals in highly reverberant environments, cepstral analysis has also been shown to be effective in machinery diagnostics, allowing us to recover impacts or other force transients from the structural vibration that results [2, 3].

To determine the effectiveness of this approach, we have evaluated it on a scale model of an actual telescope, using an actuator and a solenoid to provide a variety of excitation forces. Figure 1 shows a photograph of the truss structure serving as the scale model.

2 Background and theory

The general method we have employed for reducing the severity of repeated transient disturbances on a structure is summarized by the following steps:

1. Measure or otherwise determine correction actuator to observer Transfer Function(s) (e.g., the TF(s) from voltage sent to an amplifier powering the correction actuator to acceleration measured at an observer location(s) on the structure), then cepstrally smooth these acceleration/voltage TF(s).
2. Operate the disturbance mechanism by itself, measure and record vibration response at observer location(s), then cepstrally window the vibration signal(s).

3. Calculate a correction voltage waveform to send out to the correction actuator power amplifier, via inverse filtering of the cepstrally windowed vibration pulse(s) obtained from Step 2 with the cepstrally smoothed TF(s) obtained from Step 1. The result will in effect be a (cepstrally windowed) “source” voltage pulse, designed to produce an equal but opposite vibration response to that observed in Step 2 above.

4. Use triggering or other means to operate correction actuator the next time a disturbance occurs, observe and determine reduction in vibration.

5. Employ “Adaptive Optimization” to revise the correction voltage for subsequent disturbance events, based on the residual vibration observed in Step 4.

This method has application in situations where the disturbances are “transient” (e.g., where the duration of each disturbance force pulse is small compared to the periodicity of their occurrence, and much less than the decay time of the structural response), the location of the disturbance is known, and the time at which the disturbance operates is known (e.g., it is periodic, a local sensor detects onset, and/or the disturbance is the result of a command).

The next sections provide more detail on the cepstral smoothing, correction voltage calculation and adaptive optimization aspects of the method.

2.1 Cepstral smoothing

The cepstrum is found by first taking a Fourier transform of a signal. We take the natural logarithm of this function - yielding log magnitude for the real part and (unwrapped) phase for the imaginary part - and then take the inverse Fourier transform of that. The resulting time domain signal is the complex cepstrum. We now multiply the cepstrum with a short time window, so as to keep the “low time” portion of the cepstrum while zeroing out the “high-time” portion, resulting in smoother frequency domain log magnitude and phase functions. TFs smoothed in this way will generally be less sensitive to small variations in modal resonance frequencies and shapes [1].

The degree of cepstral smoothing is controlled by the type and length of the time window applied to the complex cepstrum. We have used a Hanning window for this, the length of which is governed by the overall duration of the disturbance force pulse. Using a Hanning function for the cepstrum window, the length of the window needs to be at least about twice the duration of the disturbance in order to accurately estimate the voltage waveform to be sent out to the correction actuator. The cepstrum window length can be made longer than this, but eventually we start to lose the robustness advantage that the smoothing gives us. Figure 2 shows the effect that cepstral smoothing has on the magnitude and phase values of a typical TF measured on the truss structure (using a Hanning window with an effective length of about 50 ms).
2.2 Estimation of correction voltage

The transient disturbances that this approach is designed to deal with may have a simple waveform of force or motion that has a frequency spectrum that will excite a large number of the structural modes, leading to a complicated response. It is this structural response that the system has to use to determine the signal to be sent to a correction actuator. The force or motion that the correction actuator must provide will mimic the transient disturbance generally in time duration and waveform.

Estimation of the correction waveform is fundamentally a source recovery problem, in which the system transfer function(s) and response(s) are known, but the source is unknown. In this case, the response measured is due not to the correction source we wish to recover, but to a separate, disturbance source. We wish to determine the “best” source waveform that will produce the observed response, so that it can be sent out to the correction actuator the next time this (same) disturbance occurs.

By using the TF from voltage supplied to the correction actuator amplifier, to acceleration at an observation point on the structure, we are trying to determine the voltage waveform to supply to the correction actuator that will create this same measured vibration at the observation point. If the correction actuator and disturbance source are co-located or “close enough together”, then supplying the negative of the recovered correction voltage to the correction actuator should result in a force that is equal and opposite to the force supplied by the disturbance source, thus leading to a global reduction in the vibration response. But in general, the role of the correction actuator is to cancel the vibration at the observer. In some applications, depending on the configuration of disturbance and actuator, that will not be the same as canceling the disturbance force.

Estimation of the correction voltage source waveform is achieved by first cepstrally windowing the measured acceleration response(s) to the disturbance. In principle, the cepstrally windowed response is then inverse filtered (deconvolved) with the cepstrally windowed TF impulse response. In practice, we use “fast deconvolution”, in which the inverse filtering takes place in the frequency domain. For the single-input, single-output (SISO) case, this essentially amounts to dividing the cepstrally smoothed acceleration spectrum by a modified form of the cepstrally smoothed TF, then inverse Fourier transforming the result in order to obtain the estimated source waveform. This is a valid approach as long as the “inverse impulse response” (i.e., that corresponding to the inverse of the modified cepstrally smoothed TF) is not time-aliased.

For the case where we wish to make use of information from multiple response locations, we can use a single-input, multiple-output (SIMO) approach to mitigate the effect of uncertain TF zero values. In this approach, we seek a least-squared-error solution for the correction voltage, \( V(\omega) \), to the over-determined system of equations described by

\[
A(\omega) = V(\omega)H(\omega),
\]

at each frequency sample, \( \omega \). Here, \( A(\omega) \) denotes a column vector holding the (cepstrally smoothed) acceleration spectra for the two response locations, and \( H(\omega) \) is a column vector containing the (cepstrally smoothed) transfer functions from the voltage source to the acceleration responses at the two locations. An optimal solution to (1) is given by

\[
V_{\text{est}}(\omega) = H^+(\omega)A(\omega),
\]

where \( V_{\text{est}}(\omega) \) is the estimated correction voltage, \( H^+(\omega) \) denotes the full rank “pseudoinverse” of \( H(\omega) \), and where rank is defined as the number of singular values larger than some tolerance value governed by the computing platform. The computation of \( H^+ \) is based on the singular value decomposition of \( H \), where any singular values less than the tolerance value are ignored by treating them as zero. The use of \( H^+ \) means that \( V_{\text{est}} \) can only be determined in such a way as to minimize the error between \( V_{\text{est}}H \) and \( A \), the true left-hand side of (1). The error minimization that we are using is based on the least-squares (Euclidean) norm, which minimizes the mean square error between the true \( A \) and the value computed by \( V_{\text{est}}H \).

2.3 Adaptive optimization

The final step in the process involves updating the current estimate of the correction voltage, based on the residual vibration observed during an attempted control event. The resulting updated correction voltage is then sent out the next time the disturbance event occurs. We refer to this step as Adaptive Optimization (AO). Using this approach, the TF is measured (or otherwise estimated) only once, followed by a single measurement of the uncontrolled disturbance. All subsequent measurements consist only of the vibration with control attempted.

While adaptive optimization of the correction voltage offers a chance to improve the vibration reduction performance the next time the disturbance occurs, the primary motivation for developing and including AO into the method arises from an expectation that the disturbance as well as the dynamic properties of the structure and correction actuator will drift over time. As these drifts occur, the control voltage must be corrected to preserve control performance. The time scale over which an improved correction signal can be calculated will be greater than the duration of a single disturbance since no adaptation can take place as the correction sig-
nal is generated and sent out. But an improved version of the correction voltage can be computed “off-line” since the vibration produced by the disturbance and the correction actuator to observer is known.

An adaptive algorithm has been designed that updates the correction voltage based on error measurements at an observation point. The resulting routine is intended to accommodate “slowly varying” changes in the disturbance or the TF, in that it relies on these quantities being constant over at least two consecutive events in order for it to adapt to a “new” disturbance or TF.

For illustration purposes, the algorithm is developed below for the SISO case, in which one TF and one observation location are considered. Also, cepstral smoothing of the TF and the vibration responses is not explicitly taken into account in this analysis.

In order to develop the analysis on which the algorithm is based, two locations are identified:

- The control location, denoted by a subscript \( c \), at which control forces are applied.
- The observation location, denoted by a subscript \( o \), at which the total acceleration is desired to be zero.

The point at which the disturbance is applied is irrelevant to this analysis, as our interest lies only in the acceleration produced by the disturbance at the observation point.

The analysis begins by applying Fourier transforms to the transient quantities, resulting in the following frequency-domain quantities:

- \( V_c \) = control voltage applied to the actuator at \( c \) (in this analysis, the term “control voltage” is used instead of “correction voltage” to avoid confusion with updates or “corrections” of certain quantities involved in the analysis).
- \( A_c \) = acceleration at \( c \) due to the control voltage applied to the actuator at \( c \)
- \( A_d \) = acceleration at \( o \) due to the disturbance
- \( G_c = A_c / V_c \) = transfer function relating control acceleration at \( o \) to control voltage

The control objective is to produce zero acceleration at the observation point, so the control error is defined as the total acceleration at the observation point:

\[
E = A_c + A_d = G_c V_c + A_d
\]  

The control voltage that produces zero acceleration (i.e., error) is

\[
V_c = -\frac{A_d}{G_c}
\]  

This calculation relies, of course, on an accurate knowledge of \( A_d \) and \( G_c \). As mentioned above, these quantities are expected to drift over time. To remedy this, we seek to correct \( A_d \) and \( G_c \) based on the error measured during previous control events. Two events are initially considered, denoted by subscripts 1 and 2, in which control was attempted and \( A_d \) and \( G_c \) are assumed to be constant over these two events. Control errors are then defined as

\[
E_1 = G_c (V_c)_1 + A_d
\]
\[
E_2 = G_c (V_c)_2 + A_d
\]  

Algebraic solution of these equations gives expressions for \( A_d \) and \( G_c \):

\[
\begin{align*}
E_1 &= G_c (V_c)_1 + A_d \\
E_2 &= G_c (V_c)_2 + A_d \\
A_d &= \frac{E_2 (V_c)_1 - E_1 (V_c)_2}{(V_c)_1 - (V_c)_2}
\end{align*}
\]  

For numerical reasons, it is advantageous to recast this result in terms of corrections to \( A_d \) and \( G_c \). Denoting the previous values by an overbar and corrections by a \( \Delta \), we define

\[
\begin{align*}
\bar{G}_c &= \overline{G}_c + \Delta G_c \\
\bar{A}_d &= \overline{A}_d + \Delta A_d
\end{align*}
\]  

and solve for the corrections

\[
\begin{align*}
\Delta G_c &= \frac{\Delta E_1 - \Delta E_2}{(V_c)_1 - (V_c)_2} \\
\Delta A_d &= \frac{(V_c)_1 \Delta E_2 - (V_c)_2 \Delta E_1}{(V_c)_1 - (V_c)_2}
\end{align*}
\]  

where

\[
\begin{align*}
\Delta E_n &= E_n - \overline{E}_n \\
\overline{E}_n &= \overline{G}_c (V_c)_n + \overline{A}_d
\end{align*}
\]  

These corrections are used to calculate a modification (“correction”) to the control voltage, \( \Delta V_c \), by writing

\[
V_c = \overline{V}_c + \Delta V_c = -\frac{\bar{A}_d}{\bar{G}_c}
\]  

so that the modification is

\[
\Delta V_c = -\frac{\bar{A}_d + \Delta \bar{A}_d}{\bar{G}_c + \Delta \bar{G}_c} - \overline{V}_c
\]  

In terms of the measured errors, this modification is

\[
\Delta V_c = -\frac{\bar{A}_d [(V_c)_1 - (V_c)_2] + (V_c)_1 \Delta E_2 - (V_c)_2 \Delta E_1 - \overline{V}_c}{\bar{G}_c [(V_c)_1 - (V_c)_2] + \Delta E_1 - \Delta E_2}
\]  

If the normalized correction \( \Delta G_c / \overline{G}_c \) is small, then the binomial expansion may be used to derive the simplified approximation

\[
\Delta V_c \approx \frac{\Delta \bar{A}_d}{\bar{G}_c} - \left( \frac{\bar{A}_d}{\bar{G}_c} \right) \frac{\Delta \overline{G}_c}{\overline{G}_c}
\]  

in which all products of corrections have been neglected. In terms of the measured errors, this approximate modification is
In general, given two consecutive control attempts denoted by \( n \) and \( n-1 \), the governing equation can be expressed in matrix form as:

\[
\begin{pmatrix}
\Delta E_1 \\
\Delta E_2
\end{pmatrix} = \begin{pmatrix}
G_c & 1 \\
G_c & 1
\end{pmatrix} \begin{pmatrix}
V_{c_1} \\
V_{c_2}
\end{pmatrix} - \begin{pmatrix}
\Delta E_1 \\
\Delta E_2
\end{pmatrix} \begin{pmatrix}
A_d \\
A_d
\end{pmatrix}
\]

Although the above equation is written for the SISO case, it is relatively straightforward to extend this form of the equation to the SIMO case, where multiple observation points are included.

The resulting algorithm consists of the following steps:

1. Begin with calibrated values of \( A_d \) and \( G_c \) (first event, zero error and control voltage).
2. Attempt control and measure error (second event).
3. Recompute the control voltage using above analysis and data from the previous two events.
5. Repeat steps 3 and 4 for each new event, updating first and second events to the previous two events.

If the system is re-calibrated during operation by measuring \( A_d \) and \( G_c \) directly, the algorithm may be restarted by returning to Step 1. Although we have not done so, the algorithm could be extended to include more than two previous events by solving for the updated \( A_d \) and \( G_c \) in a least-squares sense.

If the functions \( G_c \) and \( A_d \) do not actually change between attempts, then the control voltage obtained by (10) will not change. This will produce two identical rows in the matrix of (15), and attempts to numerically solve for \( G_c \) and \( A_d \) will be ill-conditioned. We encountered this in numerical simulations where \( G_c \) and \( A_d \) were held fixed, but did not encounter it when the algorithm was applied to the actual truss structure.

### 3 Performance of the AO algorithm

This section presents examples of typical reductions obtained using a PC-based development system to cancel various types of disturbances applied to the truss structure of Figure 1. With this system, the disturbance output signal is first sent out through one of the two D/A outputs, while at the same time the response data from the truss structure is acquired on the A/D inputs. After calculating the correction voltage, the system again sends out the disturbance signal, but this time the correction signal is also sent out on the other D/A channel. Vibration response data from accelerometers is also collected during this time, concurrent with the two outputs.

Figure 3(a) shows some example results obtained by letting the adaptive optimization algorithm run for 20 controlled events (i.e., 20 “iterations”), with the disturbance being provided by sending out a 50 ms long chirp pulse from 40-400 Hz to the disturbance actuator attached to the truss structure at location #1. The correction actuator is also mounted at location 1, but on
the opposite side of the “junction box” from the disturbance actuator. The figure shows the reductions in the overall rms acceleration levels obtained for each event when locations 1 and 8 are used as the response locations for the procedure operating in its SIMO mode (with a cepstral window width of 500 samples, at a sampling rate of 5 kHz). Here, event number 1 corresponds to no optimization, where the control voltage has been calculated based on a previous “disturbance-only” event and on previously measured TFs.

Representative results for a “triangular” type of pulse acting as the disturbance are shown in Figure 3(b). As with the chirp pulse, response locations 1 and 8 were used with the procedure operating in its SIMO mode, with a cepstral window width of 500 samples. With this broader band type of disturbance, the additional reductions achieved at location 1 during adaptive optimization were slightly less than those seen for the chirp pulse. The sudden, but temporary, decrease at event 9, is likely due to a particular hardware issue that allowed an occasional sample to be “skipped” in the A/D or D/A subsystems when both A/D and D/A were being used concurrently.

The reductions achieved with AO when a solenoid mechanism was used as the disturbance device are shown in Figure 3(c). The values achieved are somewhat less than those obtained when an actuator is used as the disturbance source, and there is also more of a “spread” between the reductions obtained at the two locations. Both of these effects may be related to the potential of the solenoid impact to generate force components in directions that are not aligned with the solenoid armature motion. These additional force components, if transferred to the truss structure at the mounting point, can not be effectively cancelled by the correction actuator due to its orientation, and could thus be factor in limiting the reductions achieved with the solenoid as a disturbance device.

In the results presented above, the same nominal disturbance was used for each event. If the disturbance should change in a “slowly varying” way, however, then we expect the AO procedure to be able to update the correction voltage accordingly, so that we have effective cancellation in subsequent events. Figure 3(d) shows what happens when the triangular wave disturbance is delayed by 0.6 ms (3 samples), starting at event 6. In this case, the procedure adapts to the change in the theoretical minimum time of 2 iterations.

We can also examine the behavior of the AO procedure when it is provided with an initial “incorrect” transfer function(s). To do this, we let the algorithm start off with TFs derived from an FEA model of the truss, instead of letting it start off with the directly measured TFs. Figure 3(e) shows the result of doing this, with the disturbance being provided by the 40-400 Hz chirp pulse. Here, the reductions for the first control attempt (i.e., before AO starts) are about 7 or so dB less than what is typically achieved when directly measured TFs are used. However, by the 4th iteration the algorithm is able to achieve reductions similar to what they were when starting out with the measured TFs.

The algorithm can also be started with both TFs simply set to unity. In this case, the initial reductions are near zero, but then by the 5th iteration the reductions have converged to roughly the same final values as we normally obtain. There may be some advantage to starting out with unity transfer functions over starting out with badly estimated TFs, as the unity TFs provide signal energy at all frequencies and therefore better signal-to-noise ratio to allow for correction.

The procedure was also found to perform well if the disturbance source consisted of multiple pulses spaced close together in time, as long as the width of the cepstral window was increased beyond that used for a single pulse in order to accommodate the longer length of the total disturbance.

4 Summary

The method described here for active vibration reduction has application in situations where the disturbances are “transient”, the location of the disturbance source is known, and the time at which the disturbance operates is known (either through a command signal or a vibration-based trigger). Reductions of 10 to 15 dB were achieved on a truss structure, and this could be improved by another 5-10 dB via an adaptive optimization procedure. The AO procedure also allows for adaptation to long-term changes to the structure or in the disturbance force.

We have implemented the necessary hardware and algorithms into a dedicated embedded system which uses one of the accelerometers to initiate sending out the correction voltage, and the reductions obtained using this system are comparable to those described above in Section 3.

References

