Vocal fold motion and voice production: mathematical modelling and experiment

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A new lumped-parameter model of human vocal folds with smooth shape derived from measurements on excised human larynges is presented. The mechanical equations of motion are based on previous studies, while the aerodynamics are solved by a new finite difference scheme. The model deals with 1-D Navier-Stokes equations, fully coupled with a 2-DOF rigid body vibrating in the channel wall. To mimic the motion of real elastic vocal folds and to avoid discontinuities in the channel, the model employs smooth spline interpolation on both edges of the rigid body. The finite difference scheme allows to take into account variable flow separation point in terms of a moving boundary condition. With the help of the model it is possible to investigate subcritical vocal fold vibrations and to determine stability thresholds for various configurations. The model is also able to perform numerical simulations of supercritical vocal fold movement including impacts. A physical model of vocal folds with the same geometry as the computational one was fabricated and measured. With the flow rate given by means of a digital mass flow controller, the vibration was observed by videostroboscopy and the acoustic output measured by a sound level meter. Mainly due to simplifications in modelling of aerodynamic effects, which accompany glottal closure, the theoretical results show only qualitative agreement with the measured data so far.

1 Introduction

First lumped-parameter dynamic models of the vocal fold self-oscillations were developed already at the beginning of the seventies of the last century (Ishizaka and Flanagan, 1972) and remain to be widely used (e.g., Liljencrants 1991; Pelorson et al. 1994, Story and Titze 1995, de Vries et al. 1999). This is because they are a meaningful alternative to finite element models, where the modelling of flow-structure interaction is still very problematic and which often require enormous computer time. The aerodynamic forces in the lumped-parameter models are usually approximated by quasi-steady forces given by the Bernoulli law, and the vocal fold geometry is described by piecewise constant, or piecewise linear functions.

It is widely accepted that the vocal fold geometry is very important in determining the stability boundaries and vibratory patterns of the vocal folds. This is why it seems useful to develop a model, which would reflect accurately the shape of real vocal folds while describing reasonably the glottal aerodynamics, and which would allow to study the influence of the vocal fold geometry and various other parameters on phonation. Such a model was recently developed by the authors on the basis of their previous studies. A physical model with the same geometry was fabricated and measured to validate the results from the computational one.

2 Mathematical model

2.1 Model of the glottis

The vocal folds are modelled by a 2DOF rigid body vibrating in the channel wall (see Fig.1). Symmetrical oscillations are assumed and thus only one half of the channel is modelled.

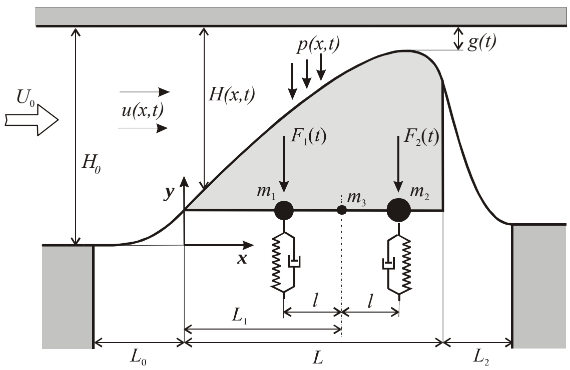


Figure 1: mathematical model of the vocal fold

The channel is considered rectangular, with uniform depth h. The function *H(x,t)* represents the time-varying channel height. In the central part $x \in [0, L]$ it is determined by the shape *a(x)* of the vibrating body and by its vertical displacement *w(x,t)* from equilibrium position:

$$H(x,t) = H_0 - a(x) - w(x,t) \tag{1}$$

whereas at the upstream and downstream extensions $x \in [-L_0, 0]$ and $x \in [L, L+L_2]$ the channel profile

$H(x,t)$ is modelled by a smooth spline interpolation. The variable $g(t)$ denotes the minimum channel height, $g(t) = \min_{x\in[-L_0, L+L_2]} H(x,t)$.

The system encounters two quite different states: first when the glottis is open – then it is necessary to calculate the velocity and pressure fields $u(x,t)$ and $p(x,t)$. In this case, the excitation forces $F_1(t)$, $F_2(t)$ are given by integration of the aerodynamic pressure $p(x,t)$. On the contrary, if the glottis is closed (the vocal folds are in contact), no airflow is present. Here, the impact forces (and possibly the static air pressure) excite the system.

2.2 Equations of motion

In the mechanical equations of motion, which are based on the previous studies (Horáček, Švec 2002), the rigid body is equivalently replaced by a three-mass system m_1, m_2, m_3 supported by springs.

The position of the vibrating element can be described by two generalized coordinates (rotation and lift):

$$ {}^{\mathrm{T}}\mathbf{V} = \left[V_1(t), V_2(t)\right] = \left[\frac{w_2 - w_1}{2l}, \frac{w_1 + w_2}{2}\right] \tag{2}$$

where $w_1 = w(L_1 - l)$ and $w_2 = w(L_1 + l)$ denote the vertical displacements at the locations of m_1, m_2 respectively. Then, from the Lagrange equations, it is easy to obtain the equations of motion of the system (see Horáček, Švec 2002)

$$\mathbf{M}\ddot{\mathbf{V}} + \mathbf{B}\dot{\mathbf{V}} + \mathbf{K}\mathbf{V} + \mathbf{F} = \mathbf{0} \tag{3}$$

where $\mathbf{M}, \mathbf{B}, \mathbf{K}$ are the structural mass, damping and stiffness matrices (2x2). Unlike the mass distribution, it is not easy to determine the stiffness, and particularly the damping inherent in real vocal folds. This is why a proportional model of damping, where $\mathbf{B} = \varepsilon_1 \mathbf{M} + \varepsilon_2 \mathbf{K}$, was chosen.

The vector ${}^{T}\mathbf{F} = \left(F_1(t), F_2(t)\right)$ stands for the excitation forces. It contains strong time discontinuities and implicitly depends on the deflection vector **V**, since it is given by completely different formulas during contact and non-contact regimes.

2.3 Aerodynamic forces

Within this study, a quasi-1D unsteady incompressible viscous flow model is used. In the open-glottis regime, the velocity and pressure functions $u(x,t)$ and $p(x,t)$ are described by Navier-Stokes equations

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} - \nu\frac{\partial^2 u}{\partial x^2} = 0 \tag{4}$$

and by the quasi-1D continuity equation for the time-space domain $\Omega_T = [0,T]\times[-L_0, L+L_2]$

$$\frac{\partial H}{\partial t} + \frac{\partial (H u)}{\partial x} = 0 \tag{5}$$

where ρ denotes the fluid density, ν the kinematic fluid viscosity and $H(x,t)$ the channel height (see Fig.1, eqn. 1). The system is equipped with initial condition, which is computed from the steady continuity equation and from the steady Bernoulli equation corrected for viscous losses (under assumption of fully developed Poiseuille flow)

$$\begin{aligned} &\frac{1}{2}\rho u^2(x) + p(x) = \frac{1}{2}\rho u^2(x_{FS}) + \Delta p_v(x), \\ &\Delta p_v(x) = 12\,\rho\nu U_0 H_0 \int_x^{x_{FS}} \frac{\mathrm{d}x}{H^3(x)} \end{aligned} \tag{6}$$

and further with boundary conditions

$$\begin{aligned} u\big|_{x=0} &= U_0 \\ p\big|_{x=x_{FS}} &= 0 \end{aligned} \tag{7}$$

To determine the flow separation point x_{FS}, the semiempirical criterion

$$\frac{H(x_{FS}, t)}{g(t)} = \eta \tag{8}$$

was used, $g(t)$ denotes the minimum channel height (see Fig.1), η is prescribed constant. The aerodynamic excitation forces are given by integration of the aerodynamic pressure

$$\begin{aligned} F_1(t) &= \frac{h}{2}\int_0^L \left(1 - \frac{x}{l} + \frac{L_1}{l}\right) p(x,t)\,dx \\ F_2(t) &= \frac{h}{2}\int_0^L \left(1 + \frac{x}{l} - \frac{L_1}{l}\right) p(x,t)\,dx \end{aligned} \tag{9}$$

where h denotes the channel depth and the parameters l, L_1 define the positions of the two springs (see Fig.1). Since the mass of the rigid element was calculated according to the mass of the tissues in the interval $x \in [0, L]$ (one may imagine massless membranes covering the rigid body), it seems reasonable to integrate the aerodynamic pressure in (9) just over $x \in [0, L]$ too, instead of the entire interval $[-L_0, L + L_2]$.

2.4 Contact forces

During the contact of the vocal folds, the excitation forces are given by decomposition of the impact force F_H, which acts at the center of the contact area, into the location of masses m_1 $(x = L_1 - l)$ and m_2 $(x = L_1 + l)$. The Hertz model is used to estimate the impact force:

$$F_H = k_H \, \delta^{3/2} \qquad (10)$$

$\delta = \max_{x\in[0,L]} \left(a(x) + w(x,t) - H_0\right)$ stands for the penetration of the vocal fold through the contact plane and k_H represents the Hertz constant, which can be calculated from the material properties of the vocal folds and from their geometry.

During vocal folds contact, one may also take into account the quasistatic air pressure, which is present in the closed subglottal volume and which presses the vocal folds apart.

2.5 Numerical solution of the coupled problem

During vocal folds contact (closed-glottis phase), the equations (3,10) yield a system of two second-order ordinary differential equations. These are transformed into the system of four first-order ODEs, which are solved with 4th-order Runge-Kutta method.

In the open-glottis regime, one has to solve the partial differential equations (4,5) for the fluid velocity and pressure. In a general two- or three-dimensional case, a finite volume or finite element method for the solution of the problem in ALE formulation (since the computational domain Ω is time-dependent) would be necessary. Within the quasi-1D approximation, however, it is possible to employ the finite difference method, where dramatically less computer time is needed.

The space-time discretisation of the equations (4,5), stabilized by upwinding, yields an explicit 1st-order scheme for the solution of discrete velocity u_i^k and pressure p_i^k. Hence, at each time level, the velocity and pressure distribution are calculated, and the excitation forces (9) determined. This allows to proceed to the next time level with the same Runge-Kutta method, as was already described.

No theory regarding error bound estimation was developed so far. Suitable spatial step Δ and time step τ, which allow efficient yet reliable computation, were estimated by means of numerical experiment. As regards the time step of the method, an adaptive refinement was implemented in the model to pick up the moment of vocal folds impact more accurately.

2.6 Numerical values of the model parameters

A fair effort was made to relate the model parameter values to the properties of real vocal folds. Since one of the innovative concepts in this model is quite a sophisticated modelling of the vocal fold shape (within the framework of 2D approximation), a special attention was paid to supply reliable data concerning the vocal fold geometry.

The detailed information on the vocal fold shape is largely incomplete due to their inaccessibility and limited resolution of standard imaging methods such as CT and MRI. Berry et al. (2001) was the first to succeed in measuring the geometry of the inferior surface of the vocal folds on excised canine larynges using wax molds. Here we use the results of our recent measurements of excised human larynges in phonation position (Šidlof et al. 2004). Figure 2 illustrates the technique, which was used to gain the vocal fold shape near mid-membranous point.

From Fig.2-f one may see that a pure polynomial regression would hardly be sufficient to model the vocal fold shape, or would at least require high-order polynomial. This is why the shape of the vocal fold was approximated by a piecewise defined smooth parabolic curve (spline), which allows to catch the big variations in local radius of curvature. In this case, the shape was approximated by function [m]

$$a(x) = \left| \begin{array}{ll} -5.79\,x^2 + 0.72\,x + 0.0019 & x \in [0, 0.0071] \\ -870\,x^2 + 12.95\,x = 0.041 & x \in [0.0071, L] \end{array} \right. \qquad (11)$$

where $L = 8.64$ mm is the length of the central part. The other geometrical parameters (see Fig.1) were taken as follows: $l/L = 0.315$, $L_1/L = 0.5$, $L_0/L = 0.5$, $L_2/L = 0.25$, $H_0 = 7.12$ mm (for $g = 0.25$ mm) and channel depth $h = 10$ mm. From the Young modulus $E = 8$ kPa and Poisson ratio $\mu = 0.4$, the Hertz constant $k_H = 304\,\mathrm{N\,m^{-2/3}}$ (cf. equation 10) was calculated according to Brepta (1974). The air density $\rho = 1.2\ \mathrm{kg/m^3}$, kinematic viscosity $\nu = 1.58\ 10^{-5}\ \mathrm{m^2/s}$ and flow separation constant $\eta = 1.1 \div 5$ were considered. The masses $m_{1,2,3}$ and the moment of inertia, necessary for the construction of the mass matrix **M** (cf. equation 3), were calculated from the geometry. A tuning procedure was used to adjust the stiffness and damping matrices **K** and **B** in order to match the natural frequencies f_1, f_2 and 3dB half-power bandwidths Δf_1=23 Hz and Δf_2=29 Hz of both resonances with values measured on real vocal folds.

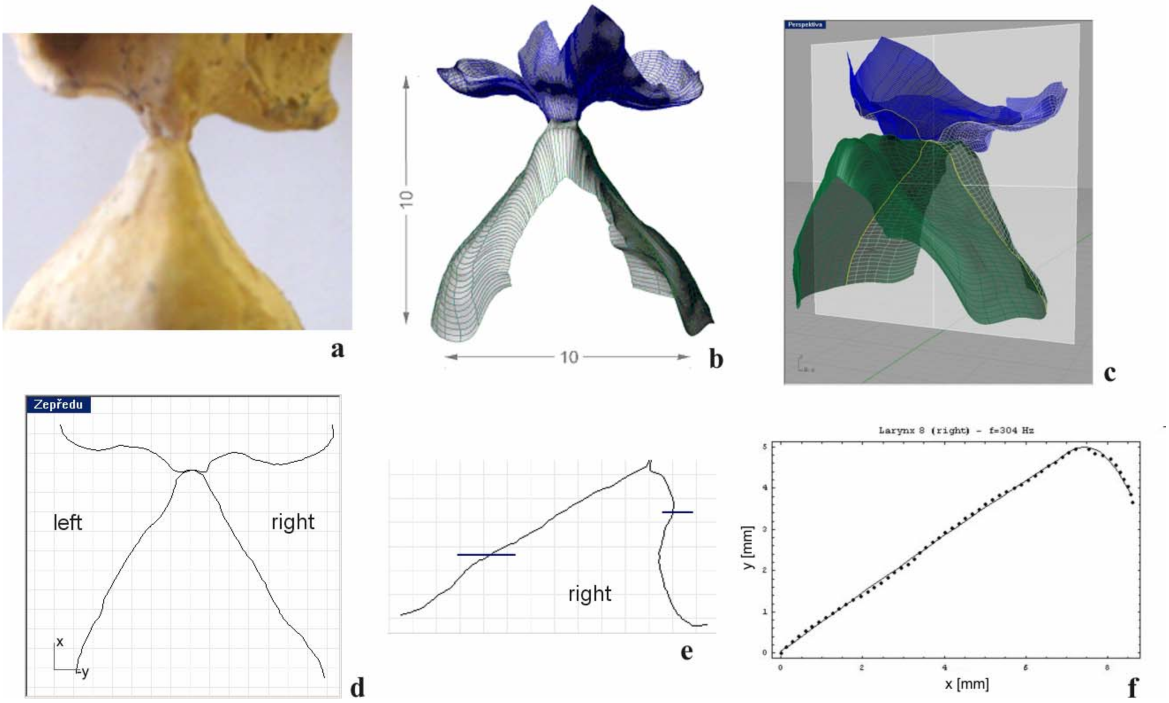
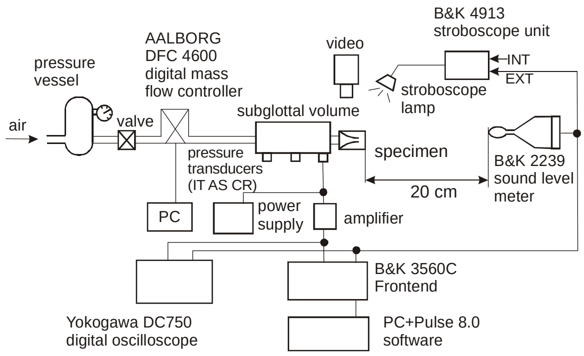


Figure 2: determination of the vocal fold geometry.
a – plaster cast of the folds which was digitized on a Wenzel LH-87 CM machine; **b** – 3D computer model; **c**, **d** – extraction of the mid-membranous coronal section; **e** – domain suitable for regression; **f** – regression curve

3 Physical model

A physical model of the vocal fold with the same geometry as the computational one was fabricated (see Fig.3). For the vibrating part, silicon rubber was used. Since the Young modulus of silicon rubber is considerably higher than the average stiffness of real vocal folds, the vibrating part was supported by two cantilever rubber beams instead of placing on massive layer. The bending stiffness of the beams was estimated so that it would approximately match the elastic properties of real vocal folds.



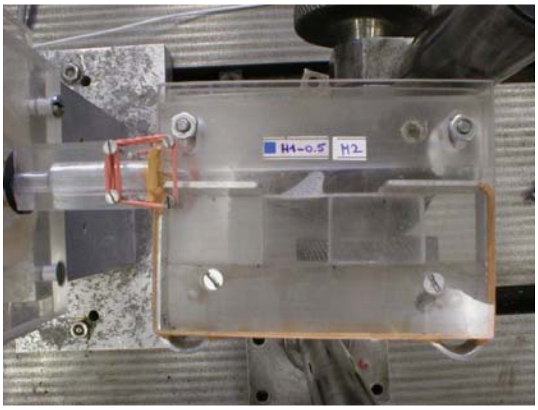


Figure 3: measurement setup and the physical model of the vocal fold

The rigid parts of the model are from plexiglass to allow videostroboscopical measurements of the vibrations.

The model of the vocal fold was mounted on a maquette of subglottal space, to which the airflow was supplied from a digital mass flow controller (see Fig.3). The vibrations were observed by videostroboscopy, and the acoustic output was measured by a sound level meter.

4 Results

For the data specified in paragraph 2.6, the mathematical model predicts critical flow velocity $U_{0,crit} = 2.25\,m/s$. For higher flow velocities, the oscillations exponentially increase until the vocal fold collides against the wall. Figure 4 illustrates the motion of the vocal fold for supercritical velocity $U_0 = 2.8\,m/s$ - after a short transient regime, the vibratory cycle stabilizes and the vocal fold exhibits regular oscillations with impacts (highlighted) in each period.

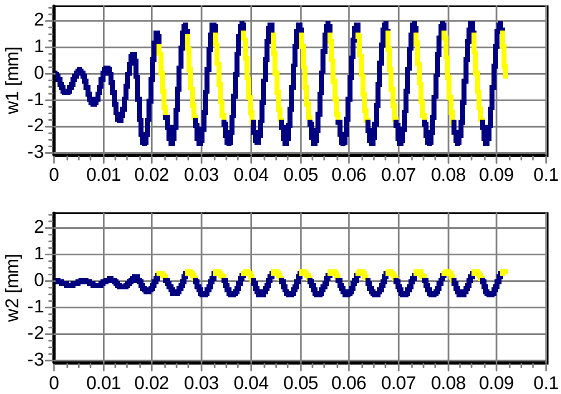


Figure 4: results from the mathematical model – deflections w1(t) (top), and w2(t) (bottom)

The model further provides plots of the glottal area and its derivative vs. time (Fig. 5), as well as several other outputs such as opening, closing and skewing coefficients, impact stress etc. These shall not be discussed in detail here, since a systematic analysis of the influence of all the input parameters is not subject of this study.

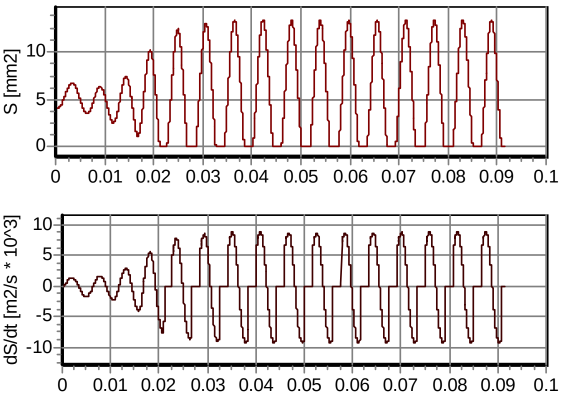


Figure 5: results from the mathematical model - glottal area (top) and its derivative (bottom) vs. time

The mathematical model also yields the acoustic pressure emitted by the vocal fold. However, the pressure may be calculated only during the open phase so far – when the glottis is closed, no flow is present and so the pressure can not be determined within this concept. Hence, one may obtain the pressure course only for vibrations without impact or for short transient start-up regimes, which is not of much practical interest.

The physical model shows qualitatively similar behaviour. When the flow rate is gradually increased, at certain flow velocity the model starts to vibrate. The acoustic pressure spectrogram is depicted on Fig. 6, where the fundamental frequency $F_0 = 170$ Hz and its harmonics are clearly visible.

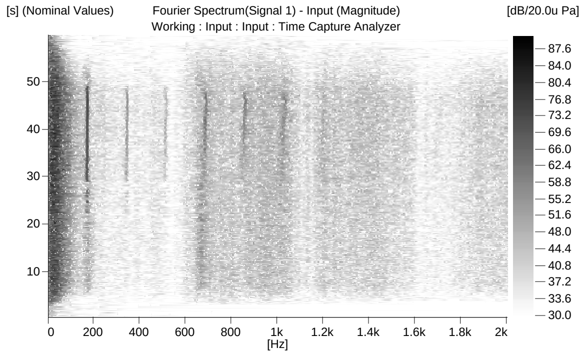


Figure 6: spectrogram of the acoustic pressure 20cm downstream from the physical model. Flow continually increased from 1.1 to 1.6 l/s

From the spectrogram one may notice that quite a strong noise was present. Moreover, the flow required to sustain vibration was too high compared to the mathematical model predictions and physiological limits. This was probably caused mainly by the leakage alongside the rubber element.

However, if one compares the videostroboscopical record of the vibration with the animation of motion generated from the mathematical model, one may state that the oscillations show the same phase difference between the upstream and downstream parts, and that the two models generally exhibit identical type of vibration.

5 Discussion and conclusions

A mathematical lumped-parameter model of human vocal fold vibrations including impacts was presented here. Compared to the previous works of the authors, several substantial new features were included: smooth channel (vocal fold) geometry without steps or discontinuous gradients, variable flow separation point and a completely new numerical scheme for the solution of the coupled problem.

Since the processes accompanying phonation are extremely complex, many simplifying assumptions still had to be imposed. One of the main drawbacks of our approach is a very rough approximation of the aerodynamic effects during glottal closure, but as far as we know, this still remains rather a challenge in other studies concerning modelling of voice production, too. Nevertheless, we tried to model the most important features in a transparent, consistent way, and to retain reasonably time-efficient algorithms, which would allow to simulate numerically the vocal fold motion and to track the influence of various input parameters.

Compared to other mathematical models known from literature, this one might be particularly useful to study the influence of geometry of the vocal folds and adjacent regions (such as the ventricular folds) on the stability boundaries and on vibratory patterns of the vocal folds.

A physical model of the vocal folds was fabricated. Due to technical problems, it was not possible to exactly reflect the construction of the mathematical model in all aspects. Taking into account the fact, that the present mathematical model can not supply the pressure field during the vocal fold contact, it was not possible to perform direct quantitative comparison so far. The subjective confrontation of the videostroboscopical record vs. the computed animation of vocal fold oscillations, however, shows encouraging agreement.

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